FOURTH PROGRESS REPORT

on

CF-105 WEAPON SYSTEM ASSESSMENT

by

R. S. Mitchell and C. J. Wilson

CANADIAN ARMAMENT RESEARCH AND DEVELOPMENT ESTABLISHMENT

Valcartier, Quebec

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PROJECT
CF-105 ASSESSMENT

FOURTH PROGRESS REPORT

Period 1 April 57 to 31 July 57

Compiled by

R.S. Mitchell &
C.J. Wilson

CANADIAN ARMAMENT RESEARCH AND DEVELOPMENT ESTABLISHMENT
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S U M M A R Y

This technical letter is a progress report on work being done at
C.A.R.D.E. in connection with the CF-105 Weapon System Assessment, record-
ing work done in the period April to July 1957.

Much effort in this period was devoted to consolidating and
analysing the work of the first year's work. These results are
reported on elsewhere. (T.M. 150/57). This report deals with three
dimensional placement work, fire control studies, missile studies and
ECM considerations that are being investigated in the second year of
the study. Preliminary results and technical discussions of these
aspects of the problem are given in the appendices.

TABLE OF CONTENTS

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1. ## INTRODUCTION

The engagement of high speed targets by supersonic interceptors armed with air-to-air missiles introduces a variety of new problems which cannot be assessed by extrapolation of data arising from experience with conventionally armed subsonic aircraft.

For this reason, CARDE has been requested by the RCAF to carry out an evaluation study of the effectiveness of a supersonic interceptor weapon system based on the AVRO CF-105 aircraft armed with Sparrow II or Sparrow III air-to-air missiles.

The primary objectives of the study as stated by the RCAF are:

(i) To evaluate the combat effectiveness of the system with different types of armament, beginning with the Sparrow series, for probable bomber threats including the Bison, Badger and Bear.

(ii) To investigate the effect of variation in fire control parameters such as A.I. radar range and look angle.

(iii) To establish the minimum acceptable level of aerodynamic performance and to investigate the effect of possible design changes in the aircraft and engine parameters, insofar as these changes affect combat performance.

(iv) To determine the effect of variations in G.C.I. placement
accuracy.

(v) To explore possible tactics and suggest optimum modes of
attack.

In order to arrive at an accurate assessment of the overall combat
effectiveness of this weapon system, the many inter-dependent sub-systems
of which it is composed require analysis, first individually and then
collectively, so that the relative importance of the principal parameters
can be established. Naturally, an exploratory study of this nature is quite
involved and certainly time-consuming, if it is to be sufficiently
exhaustive to achieve the above-stated objectives. Further, the task is
rendered difficult in that very little primary information is available
on which to base investigations, as it is evident that the establishment
of such data is perhaps the primary object of the study.

The general approach then has been to adopt a range of parameters
which should encompass final characteristics, then to conduct an analysis
based on these and thus establish their validity and importance in the
particular sub-system, as well as their influence on the effectiveness of
the system as a whole. In this way overall effectiveness can be
established as a function of the parameters of individual sub-systems
and optimum design values indicated.

Although this method is elongated and somewhat tedious, an
important compensatory feature lies in the fact that the most critical
areas requiring further study are highlighted.

2. **PROGRESS OF STUDY**

CARDE Technical Letter N-47-3, May 1956, gives a review of the general interceptor-weapon problem with particular reference to the proposed CF-105 system, and sets out in some detail a proposal for the prosecution of studies to attain the objectives enumerated by the RCAF. A directive to initiate the CF-105 Weapon System Assessment Study was received on May 29, 1956, and work has continued since. Progress of the work has been described in progress reports, CARDE Technical Letters N-47-8, N-47-12 and N-47-18. An oral presentation of the work and results of the first year of study was given to the RCAF in May 1957. The material of the presentation is detailed in CARDE Technical Memorandum 150/57. Continuation of the study for a further year has been approved, to permit more intensive investigations of certain subjects. This progress report is the first on the second phase of the work. A list of topics for this second year's work is outlined in section 4.

3. **EFFORT ALLOTTED TO THE STUDY**

3.1 Manpower Allocation

The work is being carried out by specialist sections within the Wings of CARDE, under the co-ordination and direction of the Systems Group which is generally responsible for the task.

During the period under review, a total of 12 professional personnel have been engaged in the program. The degree of participation was as follows:

	Full Time	Part Time
Systems Group	1	3
"B" Wing	0	1
"G" Wing	8	5
TOTAL	9	9

Some changes have been made in contractor help due to renewal of contracts. A few openings for suitable personnel are still to be filled.

4. **ACTIVITIES** - April - July 1957.

4.1 General

The main activity of the quarter under review has been the consolidation of the work and results of the first year of study on the CF-105. This effort culminated in an illustrated presentation to the Chief of Air Staff and the Air Council in May. These results and conclusions are documented in CARDE Technical Memorandum 150/57.

The topics to be considered in the second phase of the study are as follows:

(a) Three Dimensional placement problems.

(b) Fire Control Studies.

(c) Missile Studies (End course simulation)

(d) Lethality

(e) E.C.M.

(f) Long Range Rockets

(g) Low Altitude Targets

(h) I.R. Considerations

4.2 Placement Studies

It is considered that studies relating to the two-dimensional, co-altitude aspects of the placement problem have been finished and that critical and non-critical parameters have been defined. Work is now in progress to extend the work to three dimensions. The aerodynamic and look angle circuits have been established on the computer in their final form. Preliminary results are given in Appendix A. The details of the specific problems to be investigated are outlined and the cases reported are tabulated on page 14.

4.3 Fire Control

The general problem areas to be investigated in the fire control field have been outlined. Part time help has been obtained from a senior fire control engineer from de Havilland Aircraft Co. of Canada in firming up these problems. A summary is given in Appendix B.

4.4 Missile Studies

Missile studies in this phase of the work are concentrated on the end portion of the trajectory to determine the miss distributions that could result from noise. These results will be used in lethality studies. Further enquiries are being made at Douglas Aircraft Company to determine whether more information on this subject may be available.

The work at CARDE is reviewed in Appendix C.

4.5 Lethality

Work on the operation of the engagement simulator has been delayed awaiting the arrival of technical personnel. These people have been hired by Computing Devices of Canada but have not yet been cleared.

4.6 E.C.M.

D.R.T.E. has completed a technical assessment of the E.C.M. conditions. Their conclusions on the susceptibility of the Astra I system to ECM were reported in CARDE Technical Letter N-47-18. A similar report has been issed on the missile and fuzing problems under the title

"Notes on Susceptibility of Sparrow II and Sparrow II to Electronic Countermeasures"(by Walker and Dohoo - DRTE Report No. EL 5086-2).

Work has continued on the tactical aspects of E.C.M. in the A.I. phase. This is reported on in appendices D and G.

A project for interception trials under E.C.M. conditions is presently being proposed under the name of Sprint II. The theoretical studies to back up these trials will remain as part of the CF-105 study. It is intended to issue a preliminary program of the work in September.

4.7 Long Range Rockets

The Systems Group is presently engaged in determining how much work remains to be done in assessing the long range rocket over and above what is already available in Canada. It would seem that most effort will be required in determining what errors are involved. It is not thought that the study as a whole would be of an extended nature.

4.8 Information Sources

Outside contact has been mainly with RCA Waltham. A number of liaison visits have been made on a working level to compare methods of 3-D simulation being carried out. A visit was made to AVRO Aircraft, but no radically new data was obtained.

5. PRESENT INDICATIONS

As mentioned above the general observations, trends and conclusions gleaned from the first year of work was reported on in May (TM 150/57). This report substantially represents the state of thinking at the moment. The 3-D results are quite preliminary as indicated in Appendix A.

E R R A T A

"Third Quarterly Progress Report on CF105 Weapon System
Assessment" 1 April 1957.

(a) Page 22

Appendix 'E' , the first case should read:

$$M_T = 1.5, \qquad M_I = \underline{1.8} , \qquad H = 50K \text{ etc.}$$

and not:

$$M_T = 1.5, \qquad M_I = 1.5 , \qquad H = 50K \text{ etc.}$$

(b) Page 362

The ideal approach line shown is not correct. The true line
should make an angle of $53\frac{1}{2}°$ rather than $32\frac{1}{2}°$ with the direction
of target velocity.

APPENDIX 'A'

Progress with the Three Dimensional Placement Study

by C.J. Wilson

1. Introduction

The three-dimensional simulation of the A.I. radar controlled phase
of CF-105 interceptions has been continued. Preparation of the analogue
computer model was described in earlier Progress Reports on the project
(Refs. 1, 2 and 3) and a few preliminary results were given. It is now
possible to give tables of cases which have been completed, some of the
placement probability values which have been obtained from these results
and the proposed programme for completion of the work.

Up to the date of the previous report combined-plane attacks only
had been studied, but now computer work on snap-up attacks against a
60,000 ft. altitude Mach 2.0 target has also been completed. Placement
probability values for this case will appear in the next Progress Report.

Some other changes in the analogue computer model have been made and
they are described in this appendix.

2. Method

2.1 Launch Zones

It is assumed that the missile will fly for a time t_f between
launch and collision with the target, with a mean velocity ΔV
relative to the interceptor. Thus the flight distance F, relative
to the launching aircraft, is given by $\Delta V.t_f$

In the early work the values

$$t_f = 12.88 \text{ seconds}$$

$$\Delta V = 1165 \text{ ft./sec.}$$

$$F = 15,000 \text{ ft.}$$

were used and this value of F was used in the lead-collision steering
equations mechanized in the fire-control computer. It was assumed that
the heading error allowable at launch was 10°, independent of launch
altitude and target altitude.

These values have now been replaced by

$t_f = 8$ secs.

$\Delta V = 875$ ft/sec.

$F = 7000$ ft.

and the allowable heading error has been made a function of altitude, as follows

Altitude of Launch	Allowable Heading Error
40,000 ft.	24°
50,000 ft.	15°
60,000 ft.	8°

The influence of launch zones on placement probability values had already been studied in two dimensions, but additional study of launch zones was undertaken before the above conditions were selected for the major part of the study.

2.2 Aerodynamic Performance

Aerodynamic estimates given by A.V. Roe (Refs. 4 and 5) have replaced the earlier estimates by N.A.E. as the basis for the model of CF-105 performance. These figures are referred to as AVRO 3.3 here, and give an estimated power-limited load factor capability of 1.63 at Mach Number 1.5 at 50,000 ft. altitude, compared with the earlier figure of 1.29. The aircraft simulation includes the additional thrust to be expected at high Mach Nos. as a result of compressor rematching.

2.3 Look Angle

It has been established during a visit to R.C.A. that the limits on the azimuth gimbal deflection will be independent of the elevation of the antenna, and similarly for the elevation gimbal. Thus the limiting envelope will be rectangular as shown in Figure A - 1, rather than elliptical, as assumed at the time of the previous Progress Report.
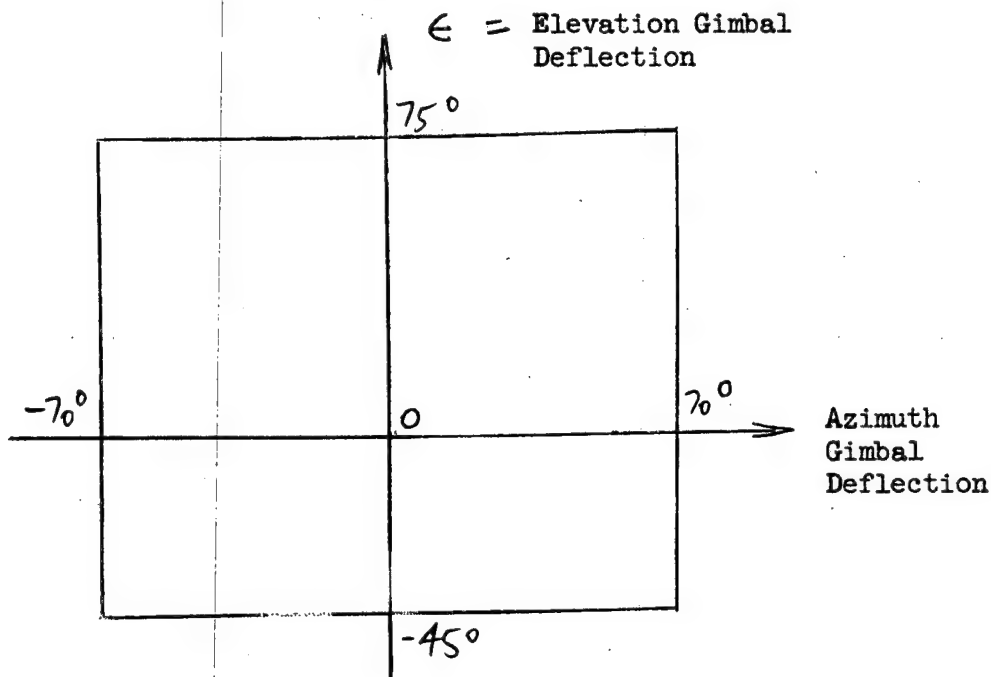
Figure A - 1

Limits of this form are now being used for the A.I. radar.

Missile blinding at launch by the airframe of the launching aircraft is very rarely a limitation on successful attack, but the effect is still included in the computer model.

It is assumed that the limits on $\in$ will be

$$- 45^{\circ} < \in < 75^{\circ}$$

but there has also been limited investigation of the cases

$$- 60^{\circ} < \in < 60^{\circ}$$

and $- 75^{\circ} < \in < 75^{\circ}$.

2.4 Snap-Up

The mechanization of snap-up which has been adopted for the simulation is simple. Initially the interceptor remains at its approach altitude, correcting the horizontal component of error only, as if the target were flying at this altitude. During this period the vertical component of error is switched out of the fire-control computer. At a given time-to-go which is a function of target altitude and of altitude difference, this vertical component is switched into the computer and from this point on the interceptor attempts to establish a true lead-collision course in three dimensions. No prediction of the decelerations which will occur during snap up and climb towards the target is made in advance.

3. Cases Considered

3.1 Computer Work Completed

Placement charts for each of the cases in Table 1 have been obtained from the computer. The work on attacks against the 60,000 ft. Mach 2.0 target is now finished. Curves of placement probability values are given for those cases where the data reduction is completed.

Throughout Problems II to IX AVRO 3.3 aerodynamics and the new look-angle limits and launch conditions are used.

TABLE I

Problem No.	Target			Max. Interceptor Mach No.	Comments
	Mach no.	Altitude (ft)	Evasion Load Factor		
0	2.0	60,000	None	2.0	N.A.E. 28% pess. aerodynamics, elliptical look angle limits, original launch zone. Results published in Ref. 3
I	2.0	60,000	None	2.0	AVRO 3.3 aerodynamics. Various look angle limits and launch zones. Designed to provide a link between problem 0 and subsequent work.
II	2.0	60,000	None	2.0	Combined plane results are given; pages to snap-up results are to be published later.
III	2.0	60,000	1.25	2.0	

3.2 Proposed Future Computer Work

The cases proposed for future study are given in Table II. Work on the 60,000 ft., Mach 2.0, target has been extensive, but each of these future targets will receive less detailed study.

TABLE II

| Problem No.	Target			Maximum Interceptor Mach No.
	Mach No.	Altitude (ft)	Evasion Load Factor	
IV	2.0	70,000	None	2.0
V	3.5	70,000	None	2.0
VI	3.5	80,000	None	2.0
VII	1.5	60,000	None	2.0
VIII	2.0	70,000	1.1	2.0
IX	2.0	60,000	None	1.5

4. ## Conclusions to Date

Study of the 60,000 ft., Mach 2.0, monmanoeuvring target has indicated that the probability of placement of the CF-105 will be high ($> 95\%$) for approach altitudes between 40,000 ft. and 60,000 ft. provided that attacks are made at near head-on aspects (course differences $> 135^{\circ}$) . In this case there is no significant advantage in the snap-up mode.

If the same target can begin an optimum conversion manoeuvre with load factor of 1.25 (0.75 g's lateral acceleration) at long enough range placement probability will be severely degraded. The target turns and runs away from the interceptor. Snap-up improves placement probability significantly in this case. Figures to substantiate this conclusion will be published in the next Progress Report.

5. ## References

1. CARDE Tech. Letter N-47-8. "First Quarterly Report on CF-105 Weapon System Assessment" Appendix G. (SECRET)

2. CARDE Tech. Letter N-47-12. "Second Quarterly Report on CF-105 Weapon System Assessment" Appendix D. (SECRET)

3. CARDE Tech Letter N-47-18. "Third Quarterly Report on CF-105 Weapon System Assessment" Appendix G. (SECRET)

4. AVRO report "CF-105 Periodic Performance Report No. 9" (SECRET)

5. AVRO report "CF-105 Periodic Performance Report No. 10" (SECRET)

PROBLEM II·A
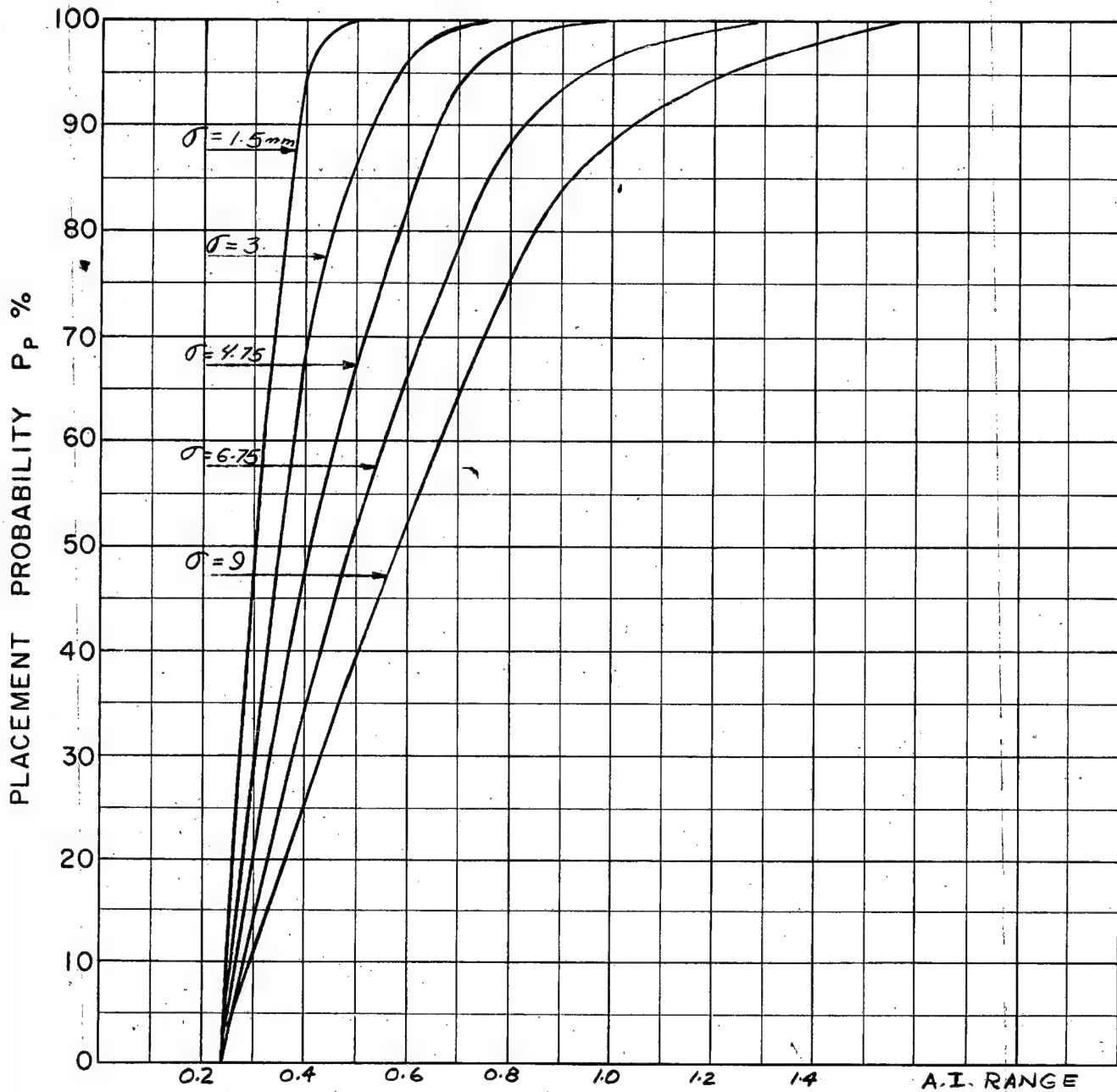
COURSE   DIFFERENCE: 180°
TARGET EVASION :   0
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's : AVRO 3.3
INTERCEPTOR   MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION   RANGE, S : Abscissa
A.I. DETECTION RANGE CONTOUR : Delta
ALTITUDE : H_t = 60 K

   H_{F_o} = 60 K

A-2

PROBLEM II B

COURSE   DIFFERENCE: 180°
TARGET   EVASION :   0
TARGET   MACH   NO.: 2.0
INTERCEPTOR   LATERAL   G's : AVRO 3.3
INTERCEPTOR   MACH   NO.: 2.0
σ OF G.C.I. ACCURACY : 5 Values
A.I. DETECTION   RANGE AS FRACTION OF SPECIFICATION   RANGE , S : Abscissa
A.I. DETECTION   RANGE CONTOUR : Delta
ALTITUDE : H_t = 60 K

H_{f_0} = 50 K

$\boxed{A-3}$

PROBLEM IIC

COURSE DIFFERENCE: 180°
TARGET EVASION: 0
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: H_t = 60 K
H_{f_o} = 40 K

(A-4)

PROBLEM IID

COURSE   DIFFERENCE: 135°
TARGET   EVASION : O
TARGET   MACH   NO.: 2.0
INTERCEPTOR   LATERAL   G's : AVRO 3.3
INTERCEPTOR   MACH   NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
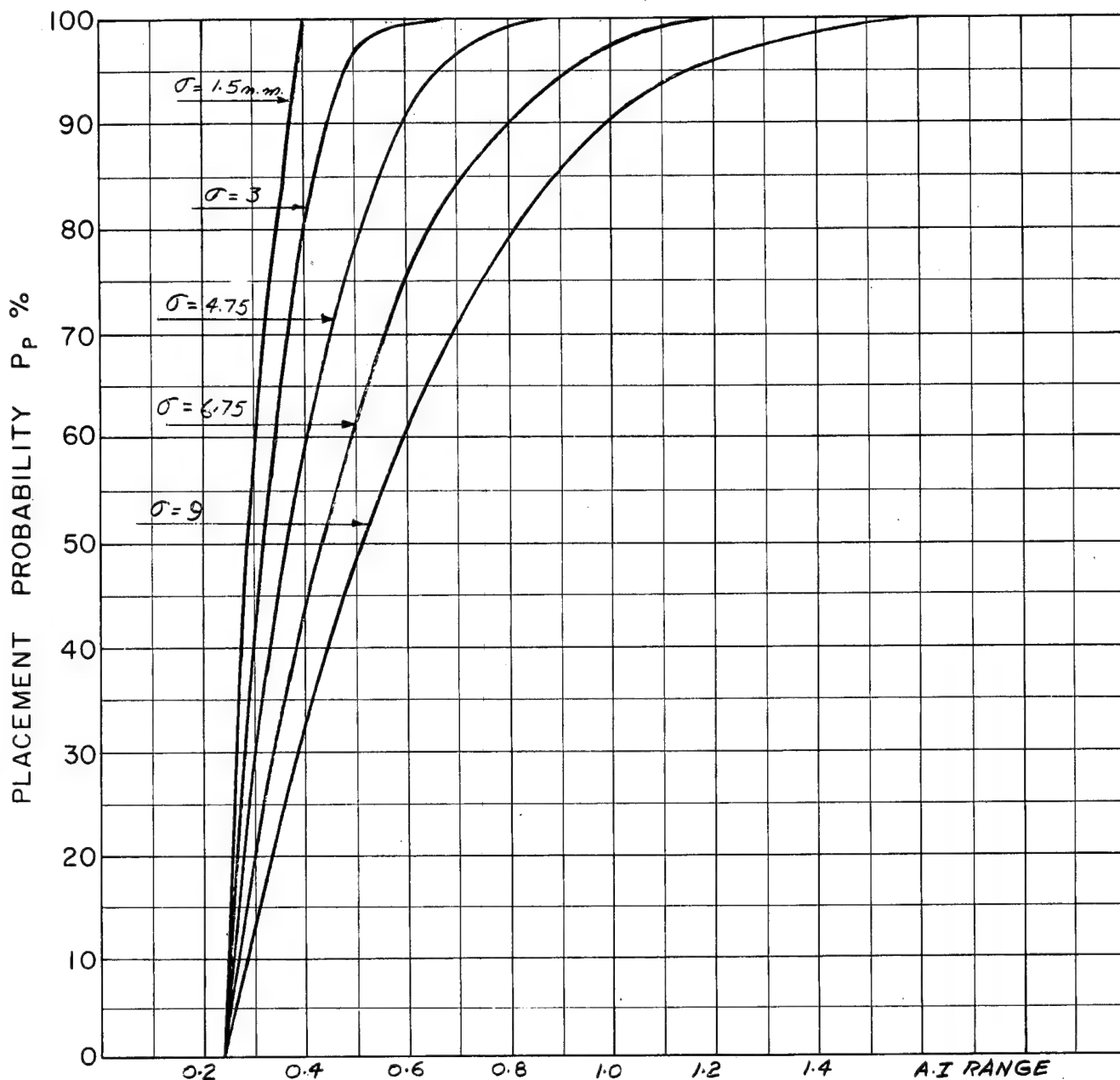A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR : Delta
ALTITUDE: $H_t = 60K$

$H_{f_o} = 60K$

A-5

PROBLEM IIE

COURSE DIFFERENCE: 135°
TARGET EVASION: 0
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: H_t = 60 K
H_{f_o} = 50 K

(A-6)

PROBLEM IIF

COURSE   DIFFERENCE: 135°
TARGET   EVASION: O
TARGET   MACH   NO.: 2.0
INTERCEPTOR   LATERAL   G's: AVRO 3.3
INTERCEPTOR   MACH   NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
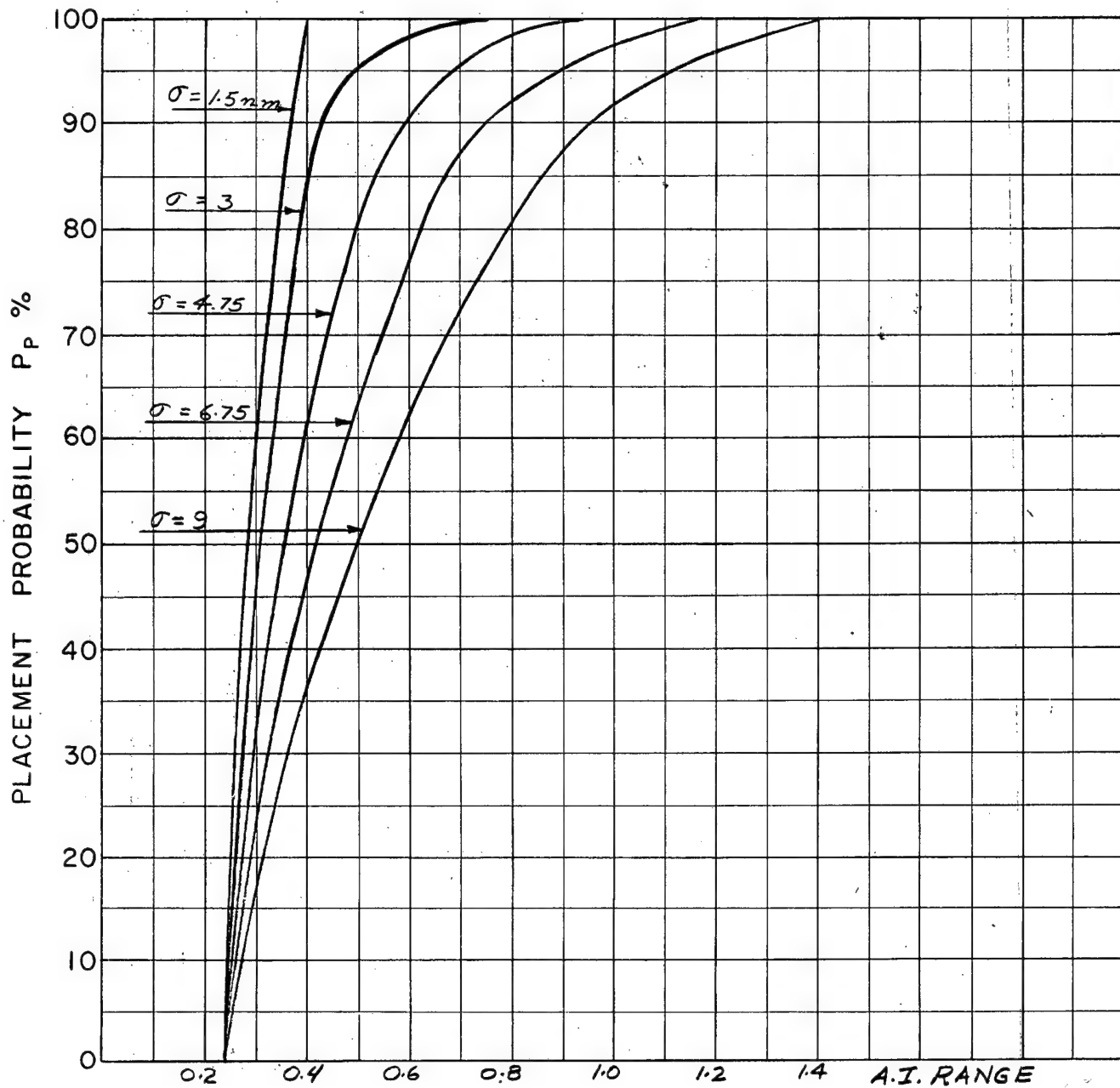A.I. DETECTION   RANGE   AS   FRACTION   OF   SPECIFICATION   RANGE, S: Abscissa
A.I. DETECTION   RANGE   CONTOUR: Delta
ALTITUDE: H_t = 60 K
                     H_{f_o} = 40 K

A-7

PROBLEM IIG

Graph: PLACEMENT PROBABILITY P_P % (vertical axis, 0 to 100) vs A.I. RANGE (horizontal axis, 0.4 to 1.4)

Curves labelled:
- $\sigma = 1.5\,n.m.$
- $\sigma = 3$
- $\sigma = 4.75$
- $\sigma = 6.75$
- $\sigma = 9$

A-8

COURSE DIFFERENCE: 110°
TARGET EVASION: O
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
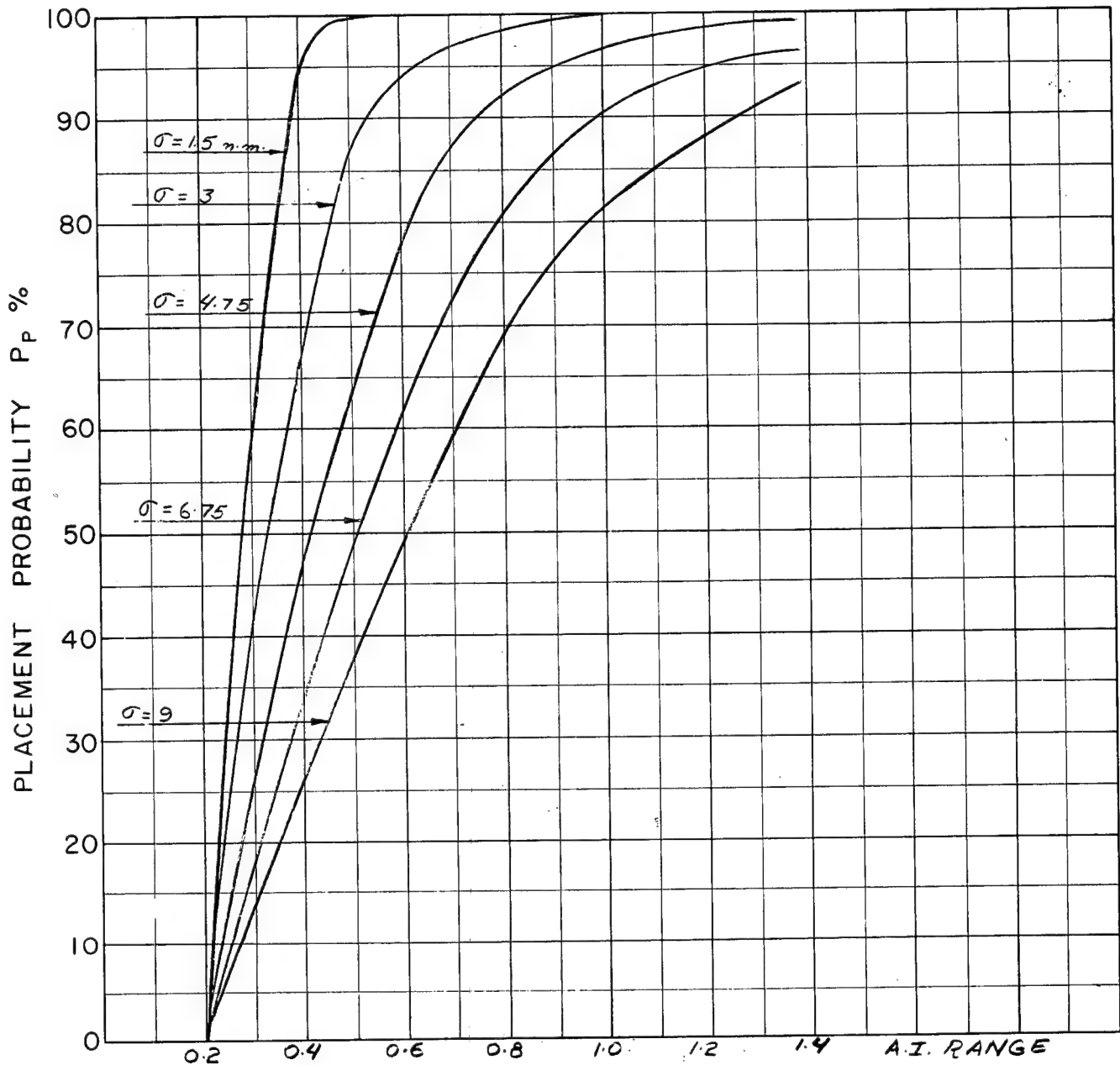A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60\,K$

$H_{f_o} = 60\,K$

PROBLEM II H

Graph: PLACEMENT PROBABILITY P_P % (vertical axis, 0 to 100) vs A.I. RANGE (horizontal axis, 0.2 to 1.4)

Curves labeled:
- $\sigma = 1.5\, m.m.$
- $\sigma = 3$
- $\sigma = 4.75$
- $\sigma = 6.75$
- $\sigma = 9$

COURSE DIFFERENCE: 110°
TARGET EVASION : ⟂
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
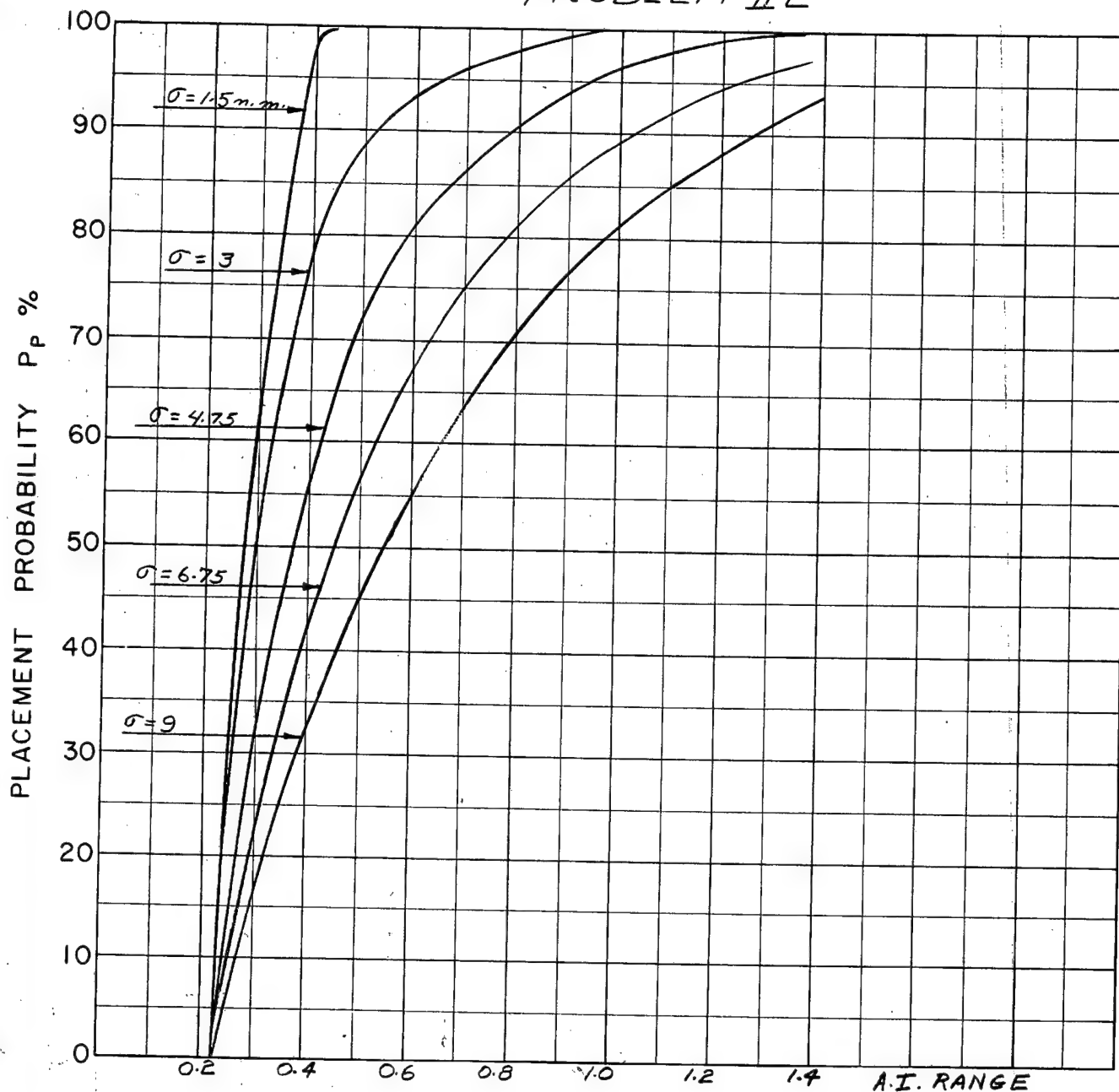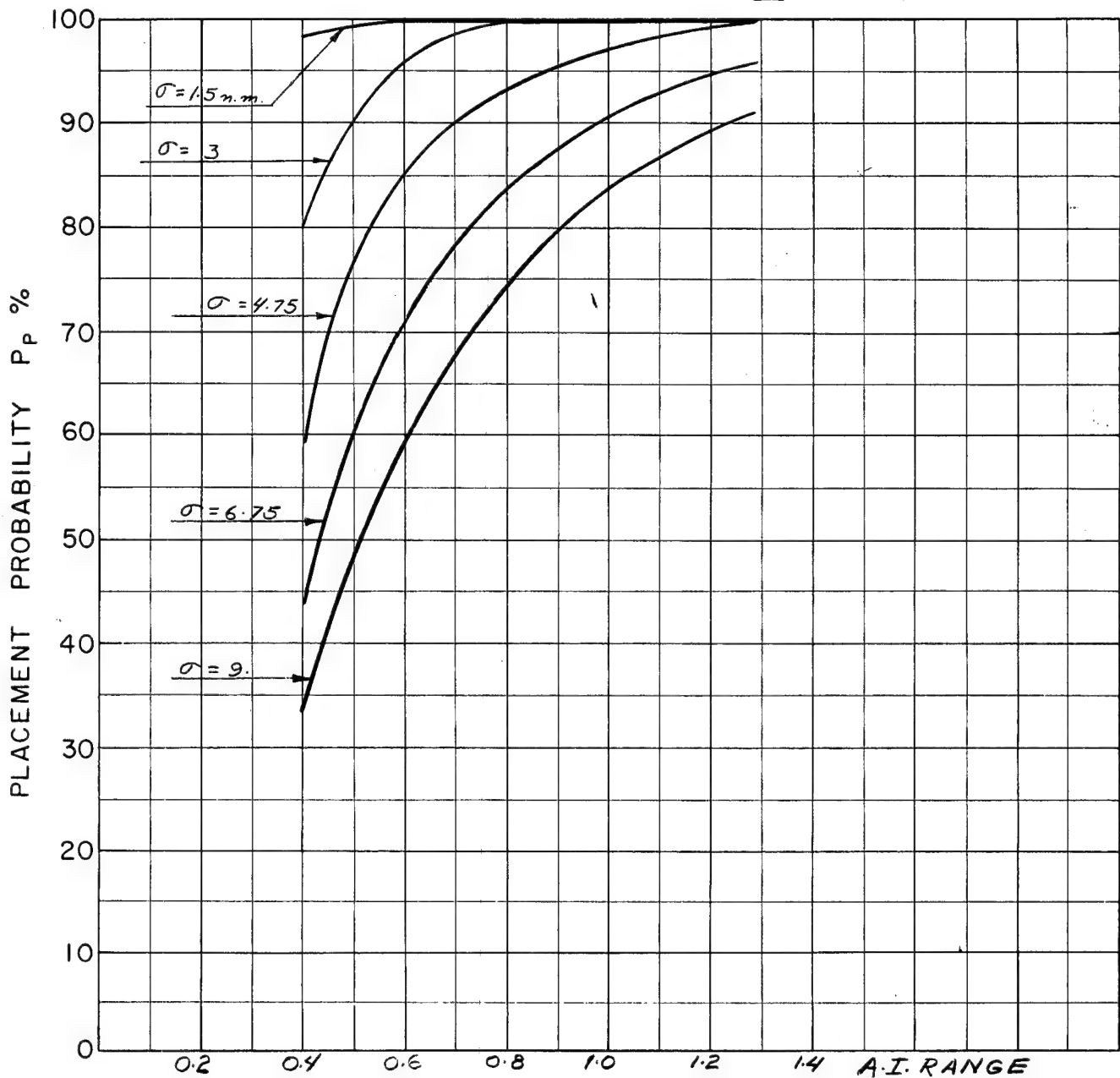A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60\,K$

$H_{f_0} = 50\,K$

(A-9)

PROBLEM IIK

COURSE   DIFFERENCE: 110°
TARGET   EVASION: 0
TARGET   MACH   NO. = 2.0
INTERCEPTOR   LATERAL   G's: AVRO 3.3
INTERCEPTOR   MACH   NO.:   2.0
σ OF   G.C.I.   ACCURACY: 5 Values
A.I. DETECTION   RANGE   AS   FRACTION   OF   SPECIFICATION   RANGE, S: Abscissa
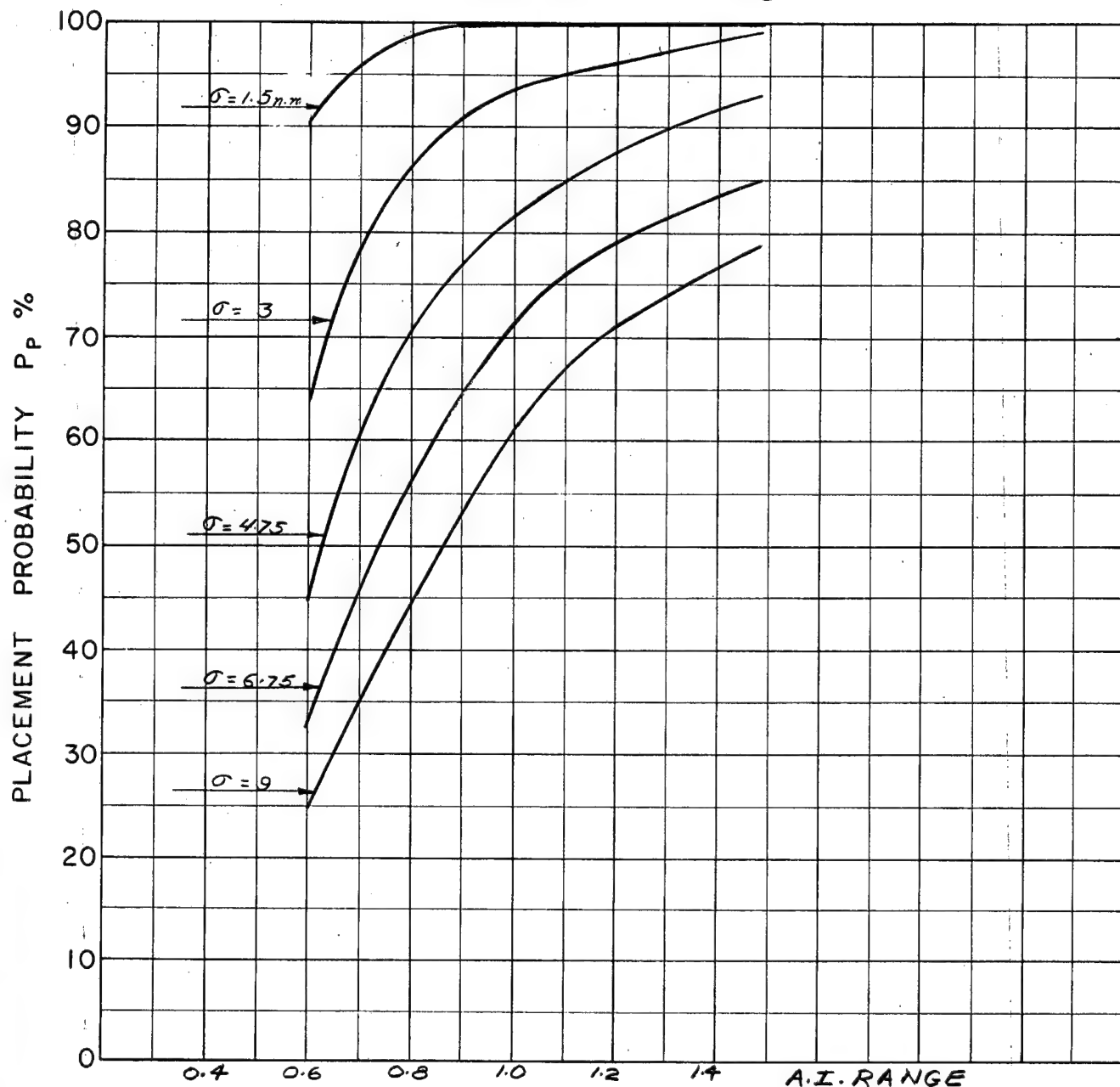A.I. DETECTION   RANGE   CONTOUR: Delta
ALTITUDE: H_t = 60 K
         H_{f_0} = 40 K

A-10

PROBLEM II L

COURSE   DIFFERENCE :   75°
TARGET   EVASION :   0
TARGET   MACH   NO. : 2.0
INTERCEPTOR   LATERAL   G's : AVRO 3.3
INTERCEPTOR   MACH   NO. : 2.0
σ OF G.C.I. ACCURACY : 5 Values
A.I. DETECTION   RANGE   AS   FRACTION   OF   SPECIFICATION   RANGE, S : Abscissa
A.I. DETECTION   RANGE   CONTOUR : Delta
ALTITUDE : H_t = 60 K

H_{f_0} = 60 K

(A-11)

PROBLEM IIM

PLACEMENT PROBABILITY P_P %

$\sigma = 1.5 n.m$

$\sigma = 3$

$\sigma = 4.75$

$\sigma = 6.75$

$\sigma = 9$

A.I. RANGE

COURSE DIFFERENCE: 75°
TARGET EVASION: 0
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
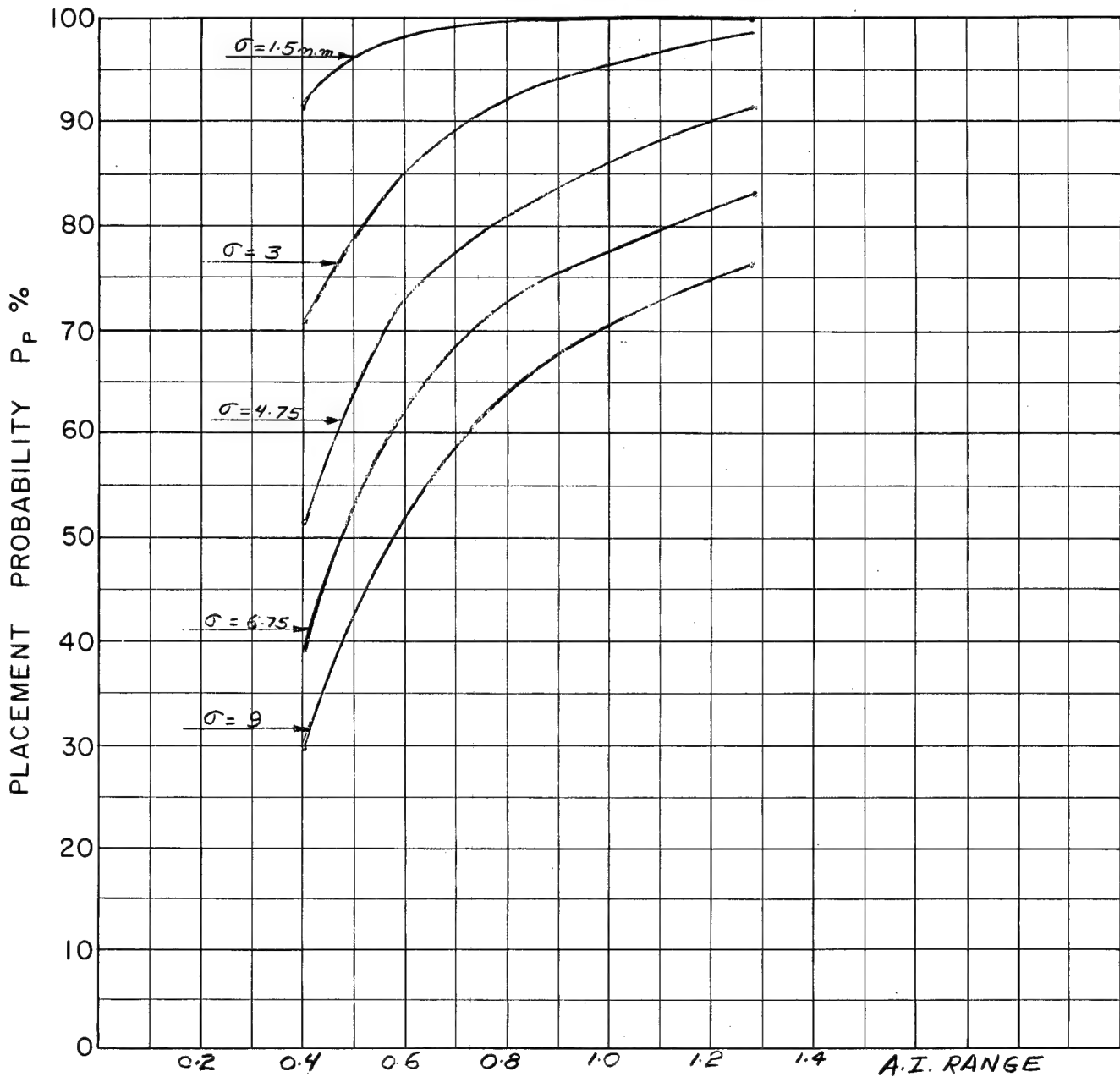σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60 K$

$H_{f_0} = 50 K$

A-12

PROBLEM IIN

COURSE   DIFFERENCE: 75°
TARGET   EVASION: 0
TARGET   MACH   NO.:   2.0
INTERCEPTOR   LATERAL   G's: AVRO 3.3
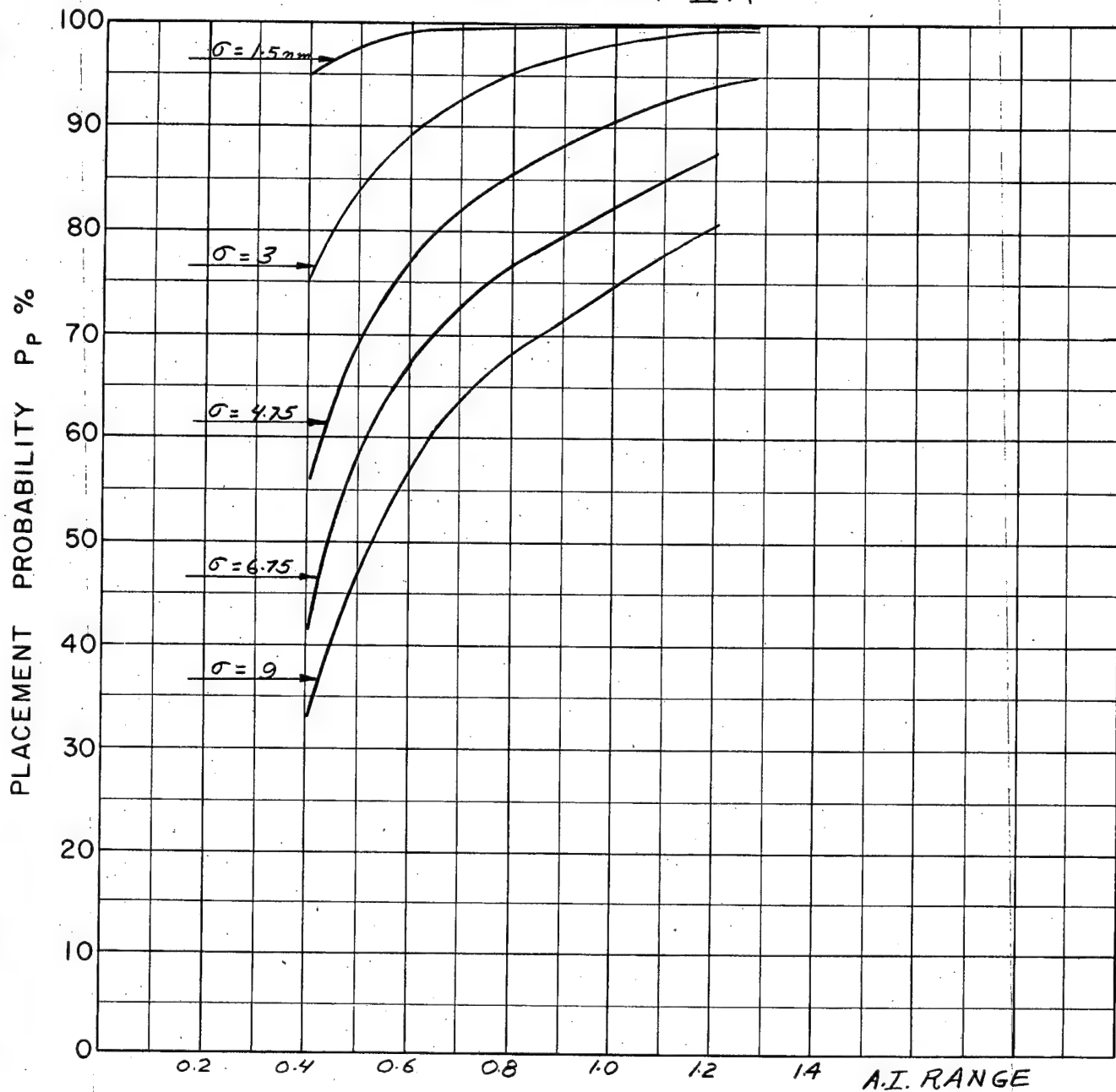INTERCEPTOR   MACH   NO.: 2.0
σ OF   G.C.I.   ACCURACY: 5 Values
A.I. DETECTION   RANGE   AS   FRACTION   OF   SPECIFICATION   RANGE, S: Abscissa
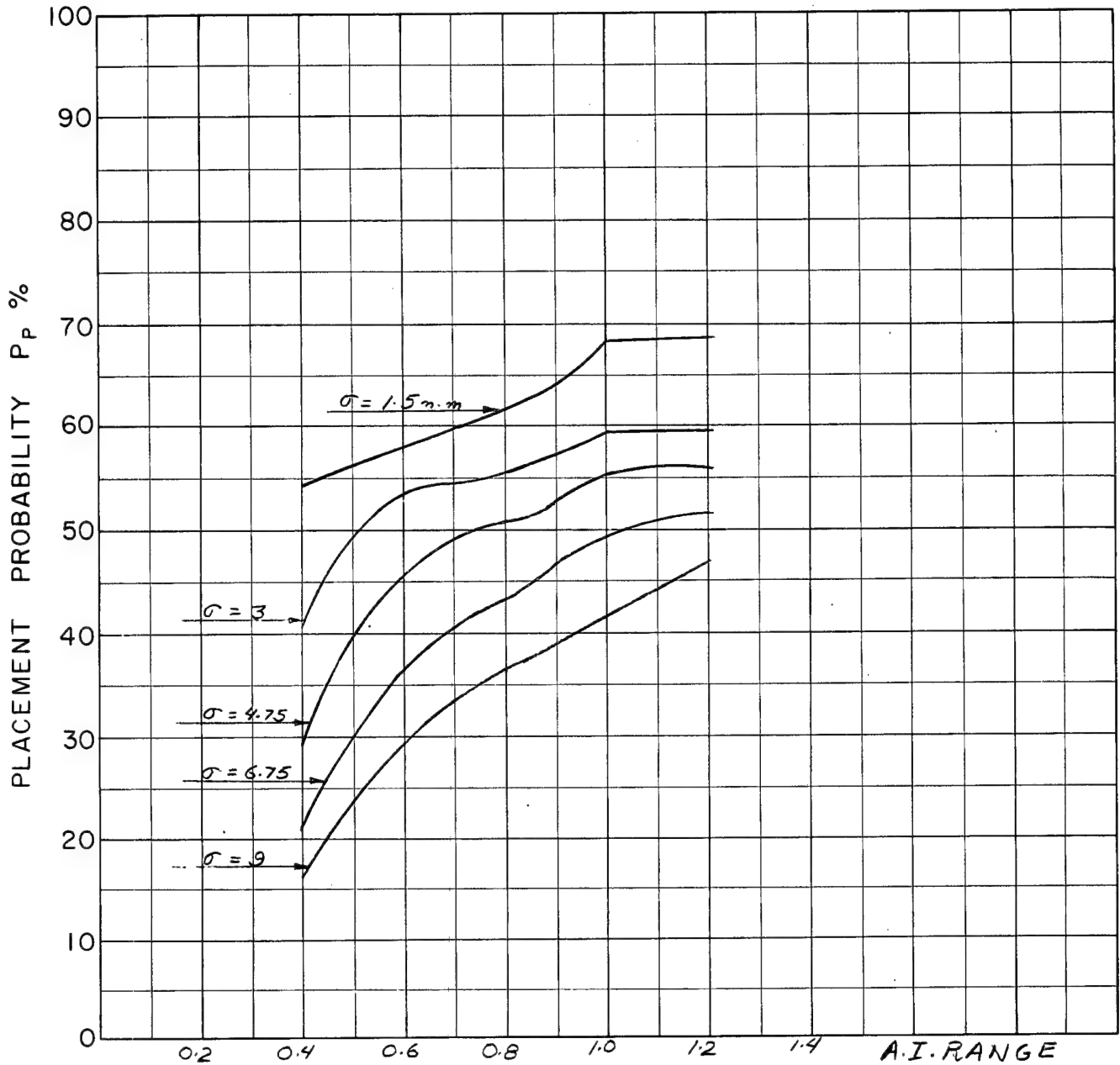A.I. DETECTION   RANGE   CONTOUR: Delta
ALTITUDE: H_t = 60 K

H_{f_o} = 40 K

A-13

PROBLEM III A

PLACEMENT PROBABILITY P_P %

A.I. RANGE

$\sigma = 1.5 nm$
$\sigma = 3$
$\sigma = 4.75$
$\sigma = 6.75$
$\sigma = 9$

A-14

COURSE DIFFERENCE: 180°
TARGET EVASION: 0.75 g. Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
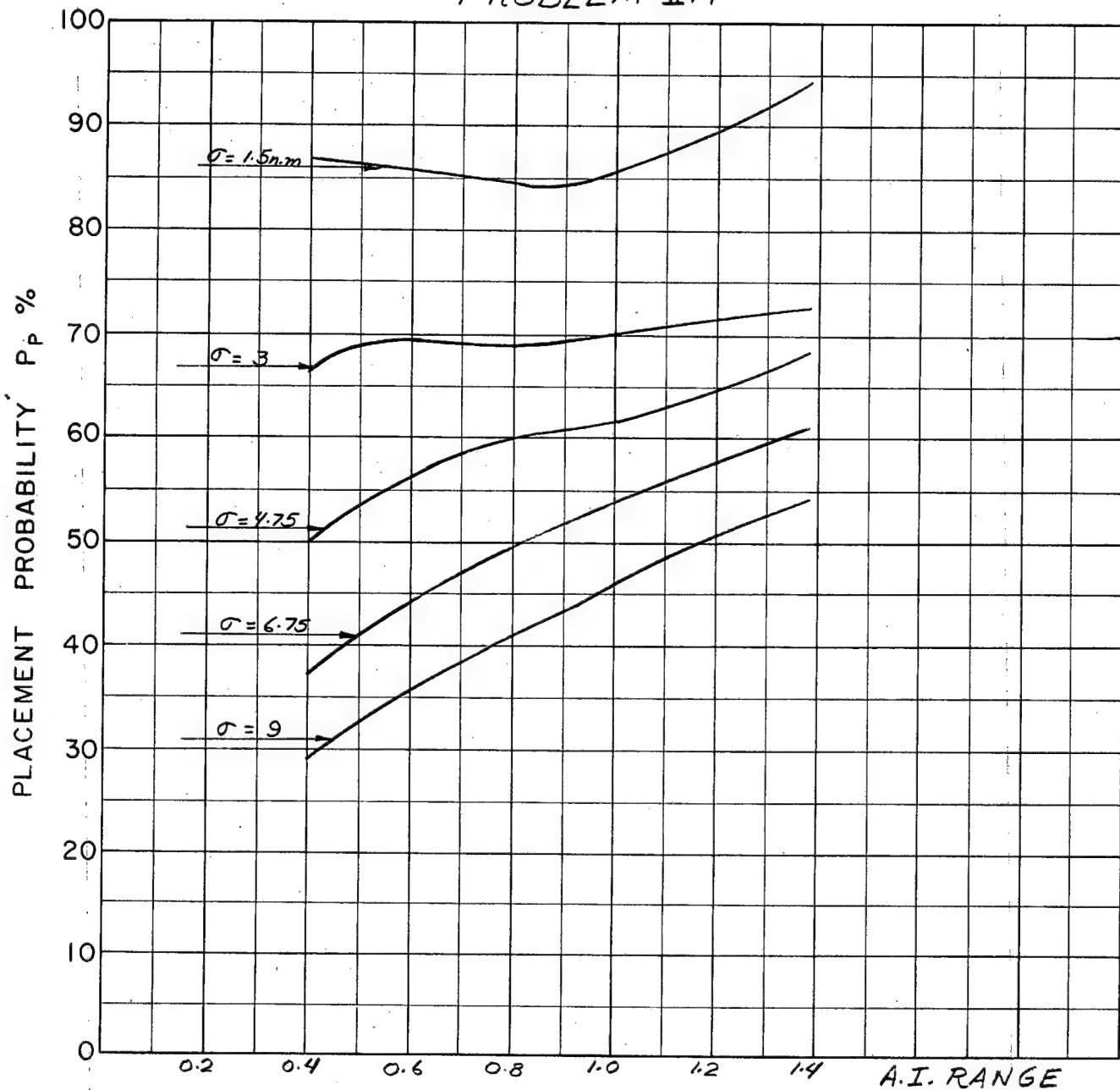ALTITUDE: $H_t = 60 K$

$H_{f_o} = 60 K$

PROBLEM III B

PLACEMENT PROBABILITY P$_P$ %

$\sigma = 1.5 nm$

$\sigma = 3$

$\sigma = 4.75$

$\sigma = 6.75$

$\sigma = 9$

A.I. RANGE

100 90 80 70 60 50 40 30 20 10 0

0.2 0.4 0.6 0.8 1.0 1.2 1.4

A-15

COURSE DIFFERENCE: 180°
TARGET EVASION: 0.75g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60K$

$H_{f_0} = 50K$

PROBLEM IIIC

COURSE   DIFFERENCE: 180°
TARGET   EVASION: 0.75 g Lateral
TARGET   MACH   NO.:   2.0
INTERCEPTOR   LATERAL   G's: AVRO 3.3
INTERCEPTOR   MACH   NO.:   2.0
σ   OF   G.C.I.   ACCURACY: 5 Values
A.I.   DETECTION   RANGE   AS   FRACTION   OF   SPECIFICATION   RANGE, S: Abscissa
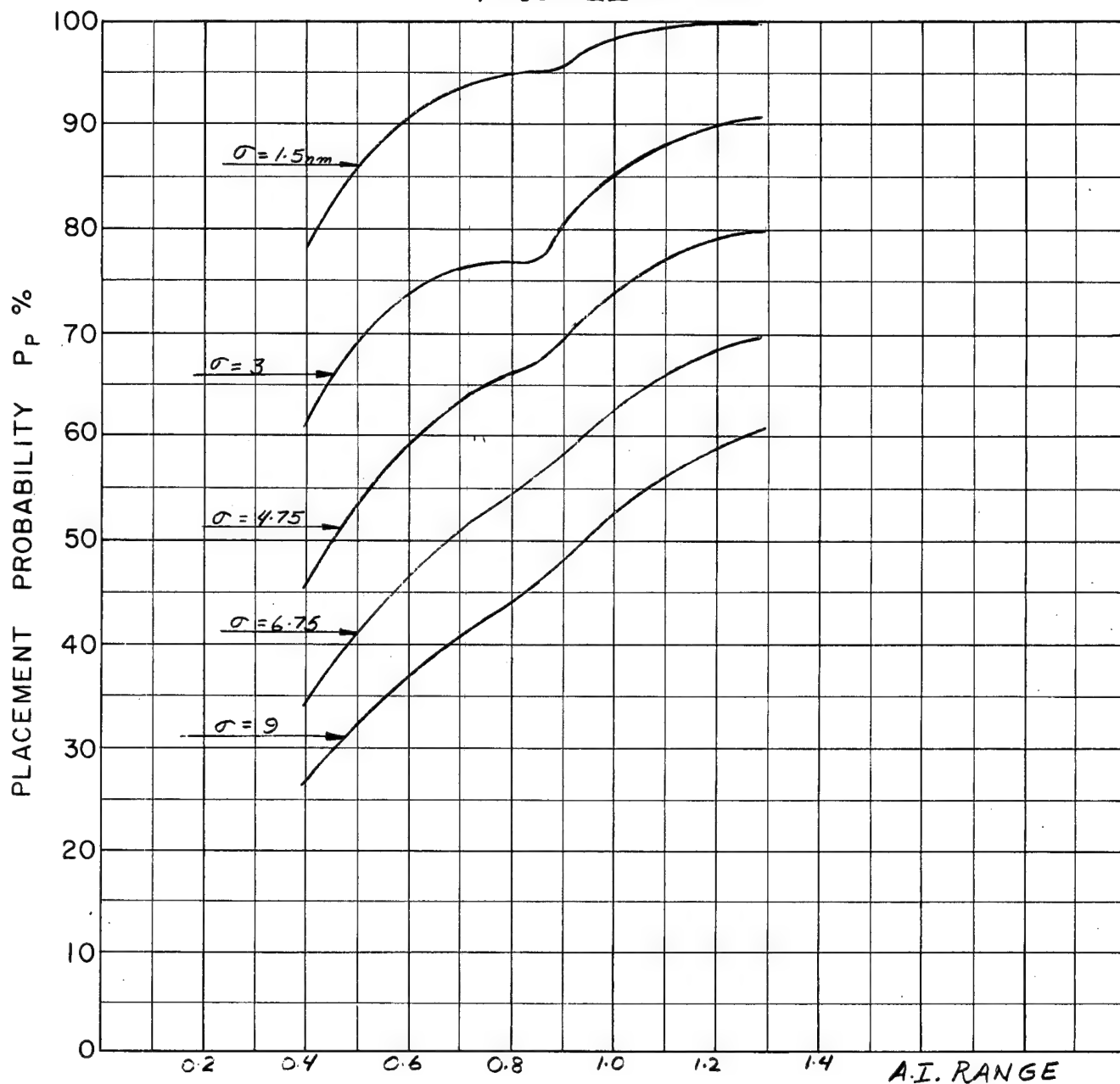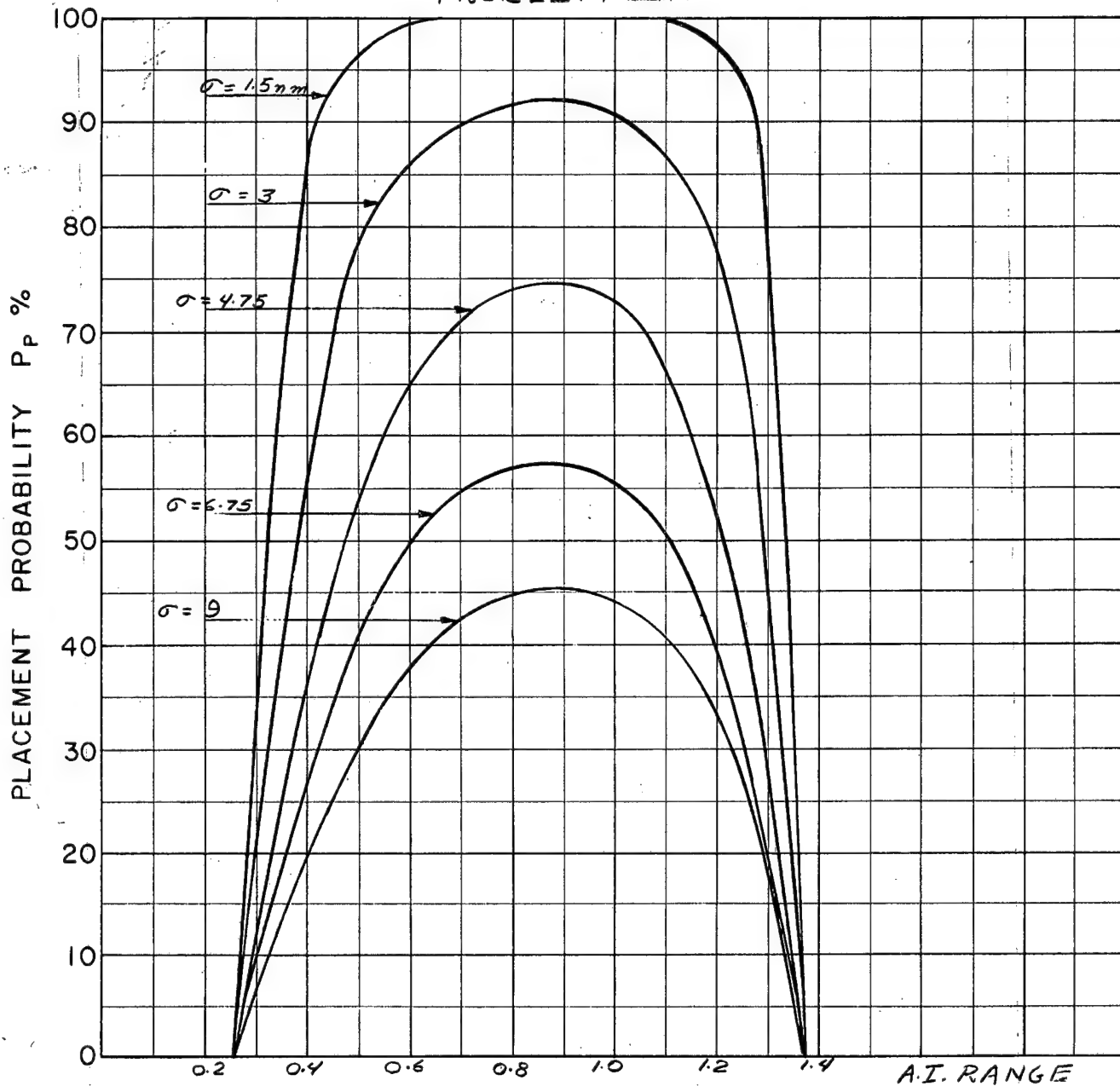A.I.   DETECTION   RANGE   CONTOUR: Delta
ALTITUDE: H_t = 60 K

H_{fo} = 40 K

(A-16)

PROBLEM III D

$\sigma = 1.5 \, n.m.$

$\sigma = 3$

$\sigma = 4.75$

$\sigma = 6.75$

$\sigma = 9$

PLACEMENT PROBABILITY P_P %

A.I. RANGE

A-17

COURSE DIFFERENCE: 135°
TARGET EVASION: 0.75g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: H_t = 60 K

H_{f_0} = 60 K

PROBLEM III E

COURSE DIFFERENCE: 135°
TARGET EVASION: 0.75 g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
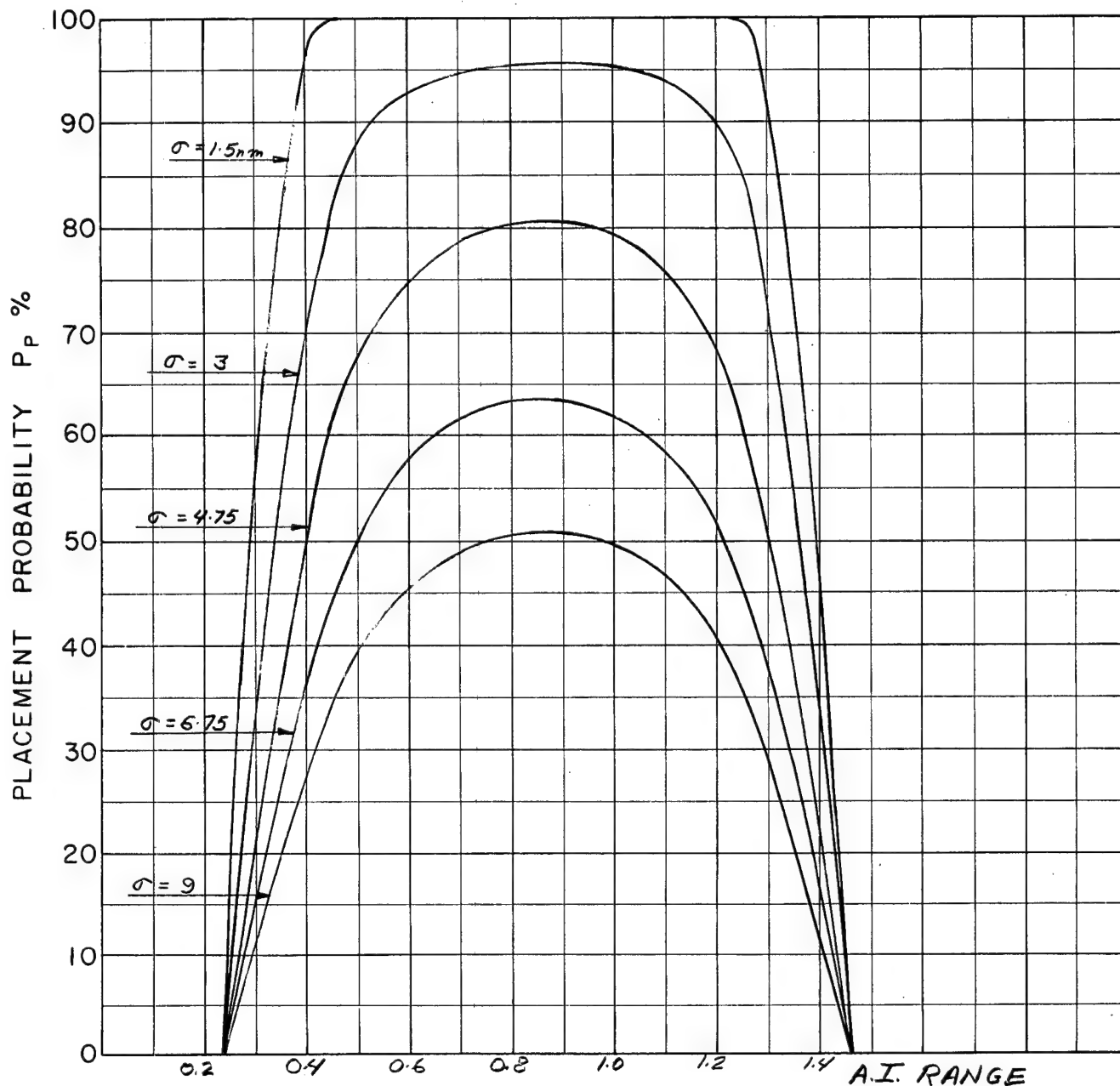ALTITUDE: H_t = 60 K

H_{f_0} = 50 K

A-18

PROBLEM III F

COURSE   DIFFERENCE: 135°
TARGET   EVASION: 0.75 g Lateral
TARGET   MACH   NO.: 2.0
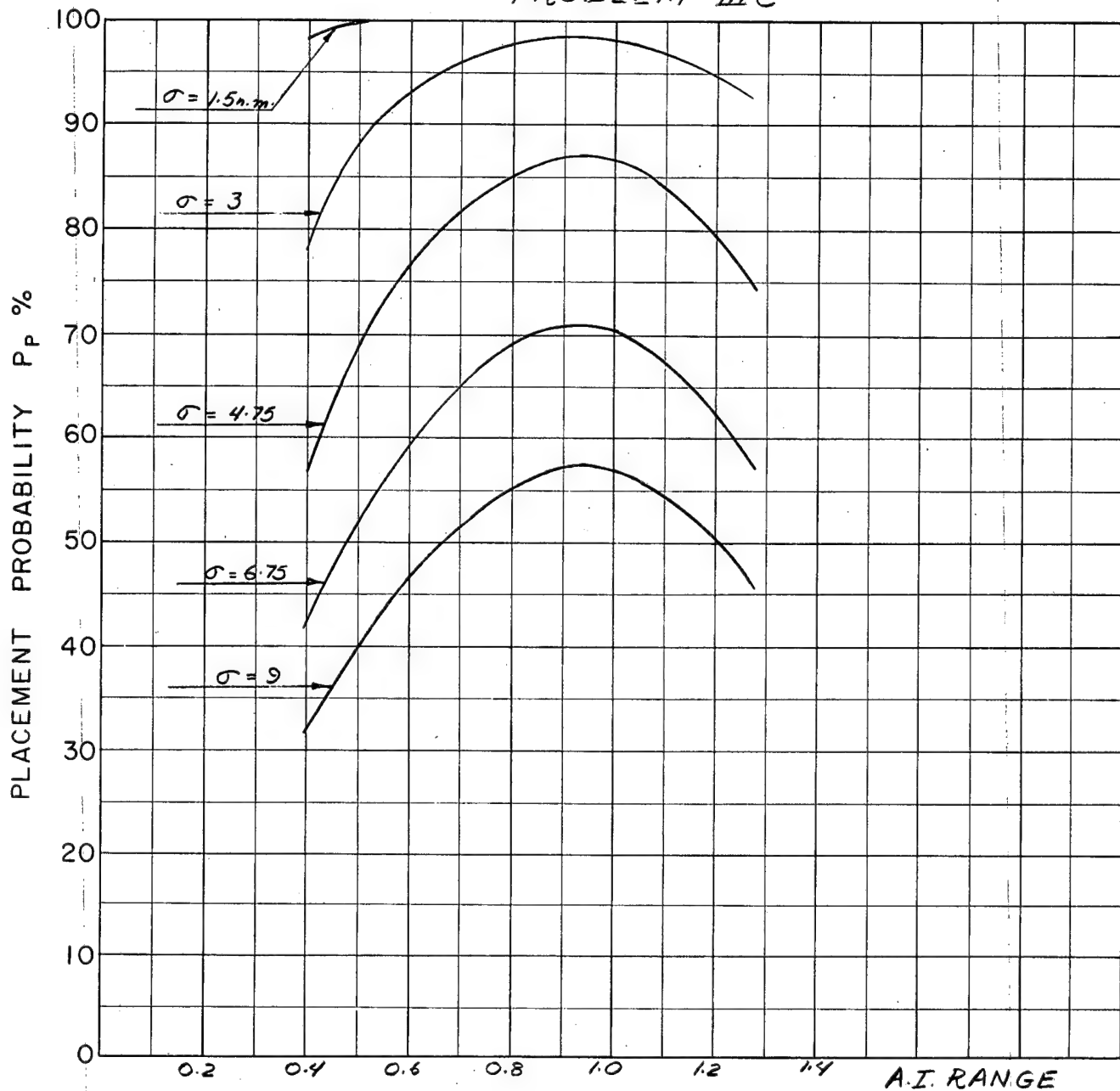INTERCEPTOR   LATERAL G's : AVRO 3.3
INTERCEPTOR   MACH   NO.: 2.0
σ OF G.C.I. ACCURACY : 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION   RANGE, S: Abscissa
A.I. DETECTION   RANGE CONTOUR : Delta
ALTITUDE : H_t = 60 K

             H_{f_0} = 40 K

A-19

PLACEMENT PROBABILITY P_P %

$\sigma = 1.5\, n.m$

$\sigma = 3$

$\sigma = 4.75$

$\sigma = 6.75$

$\sigma = 9$

A.I. RANGE

A-20

COURSE   DIFFERENCE: 110°
TARGET   EVASION: 0.75g Lateral
TARGET   MACH   NO.: 2.0
INTERCEPTOR   LATERAL   G's: AVRO 3.3
INTERCEPTOR   MACH   NO.: 2.0
σ OF   G.C.I.   ACCURACY: 5 Values
A.I. DETECTION   RANGE   AS   FRACTION   OF   SPECIFICATION   RANGE, S: Abscissa
A.I. DETECTION   RANGE   CONTOUR: Delta
ALTITUDE: $H_t = 60\,K$
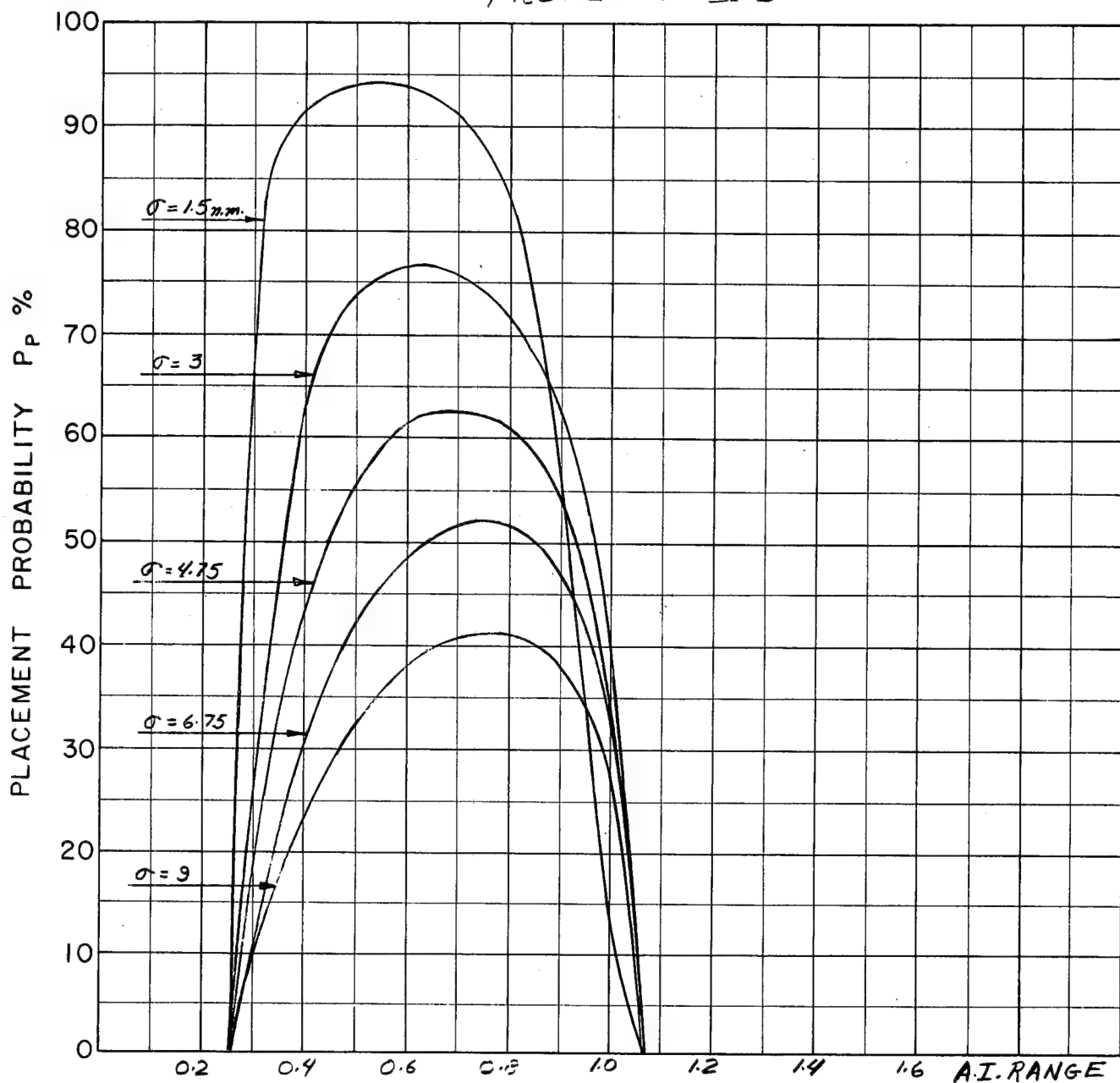$H_{f_0} = 60\,K$

PROBLEM III H

COURSE DIFFERENCE: 110°
TARGET EVASION: 0.75g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60 K$
        $H_{Fo} = 50 K$

A-21

PROBLEM III K

PLACEMENT PROBABILITY P_P %

$\sigma = 1.5mm$

$\sigma = 3$

$\sigma = 4.75$

$\sigma = 6.75$

$\sigma = 9\ mm$

A.I. RANGE

A-22

COURSE DIFFERENCE: 110°
TARGET EVASION: 0.75g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values.
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60$ K

$H_{f_0} = 50$ K

PROBLEM III M

PLACEMENT PROBABILITY P_P %

A.I. RANGE

COURSE DIFFERENCE: 75°
TARGET EVASION: 0.75g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
INTERCEPTOR MACH NO.: 2.0
σ OF G.C.I. ACCURACY: 5 Values
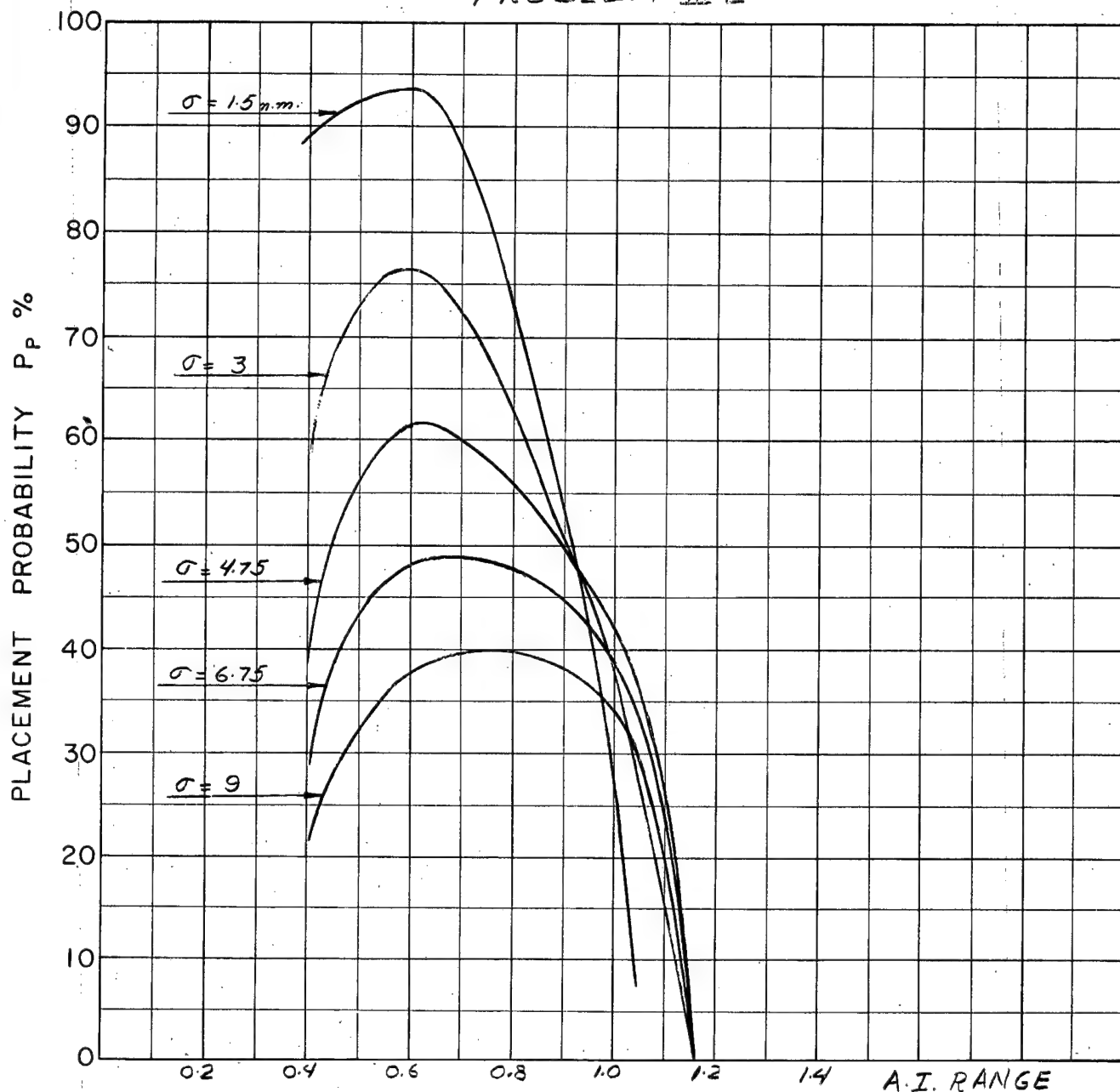A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: H_t = 60 K
H_{f_b} = 50 K

A-23

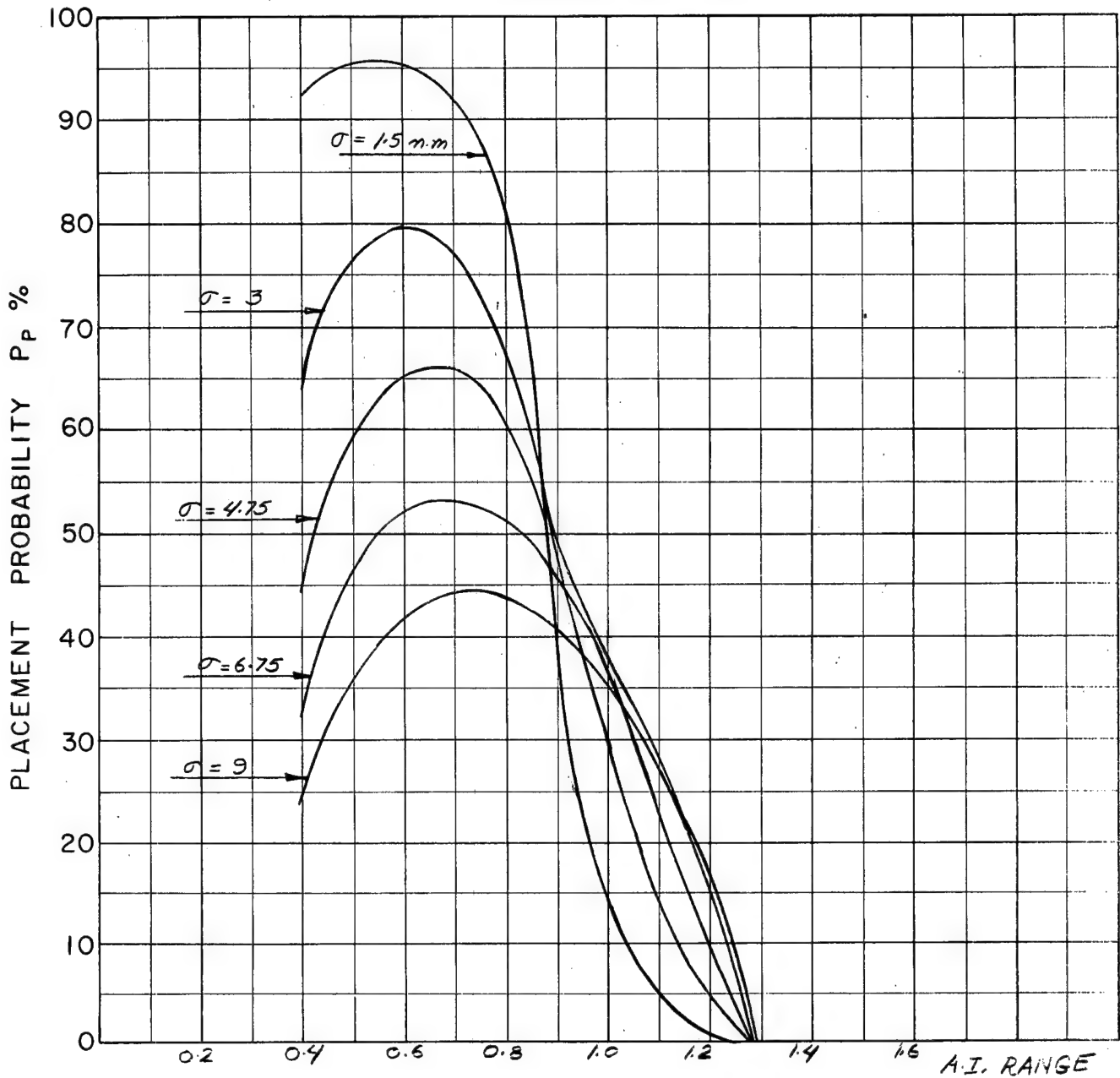# PROBLEM III N

COURSE DIFFERENCE: 75°
TARGET EVASION: 0.75 g Lateral
TARGET MACH NO.: 2.0
INTERCEPTOR LATERAL G's: AVRO 3.3
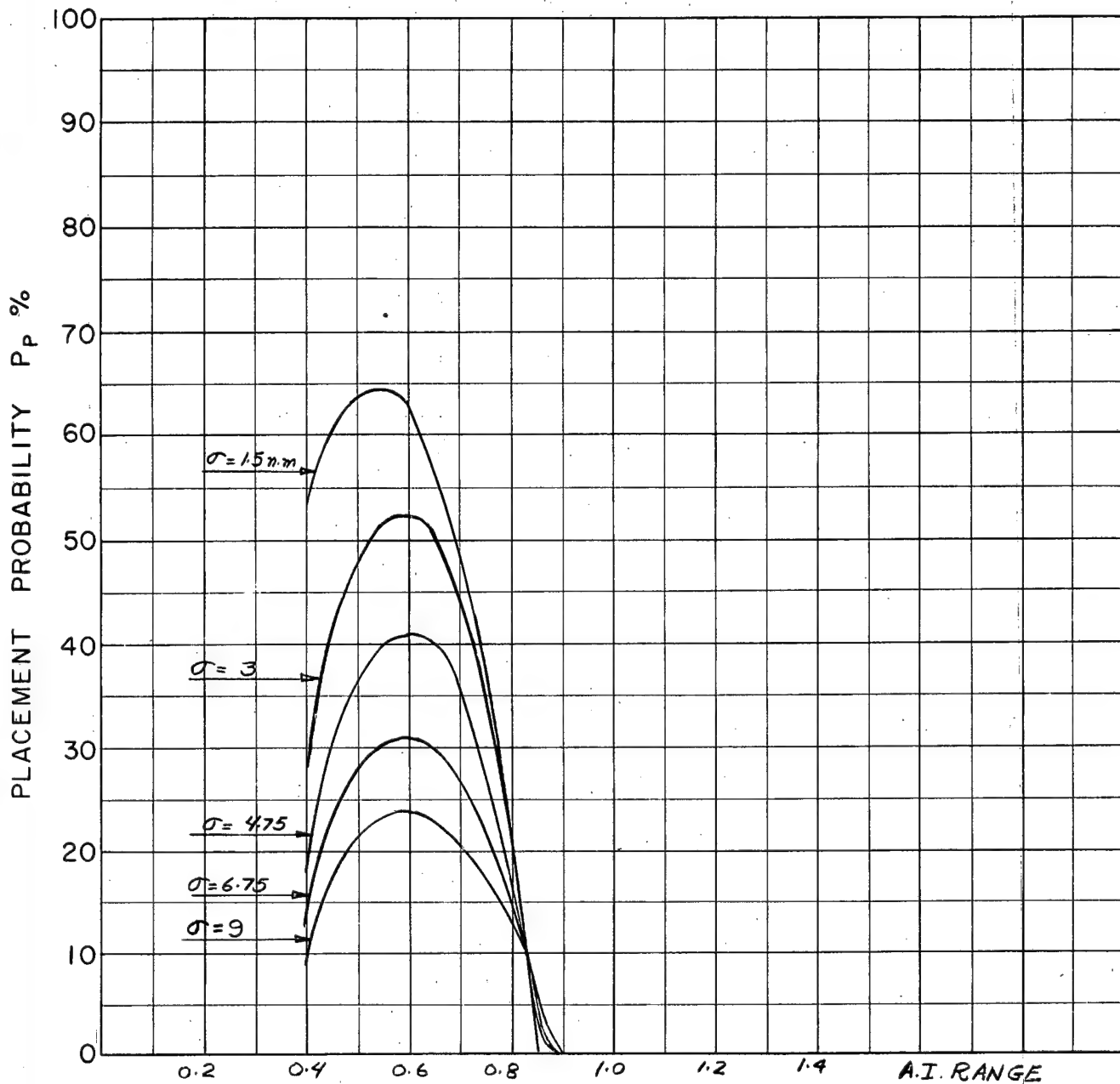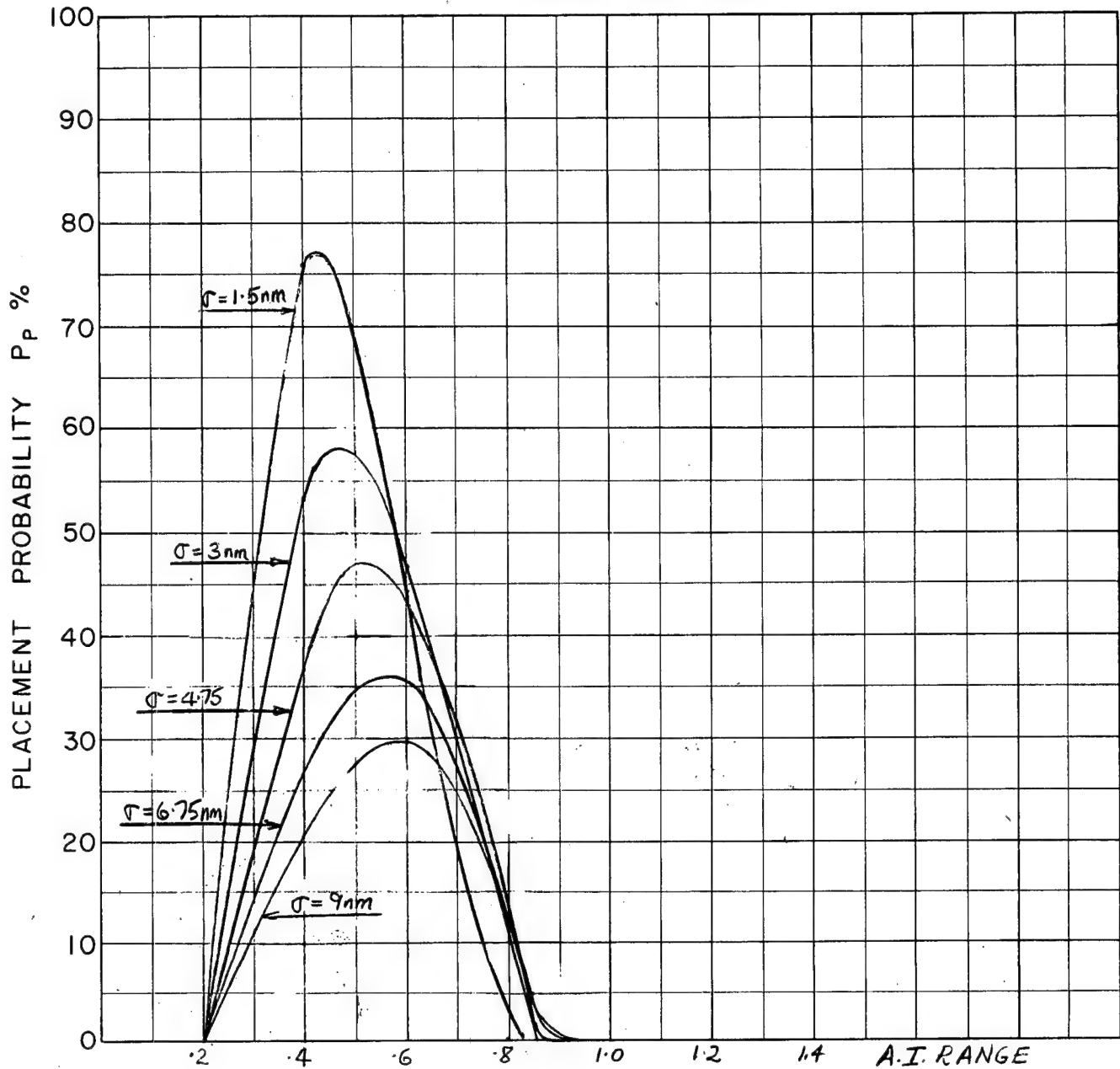INTERCEPTOR MACH NO.: 2.0
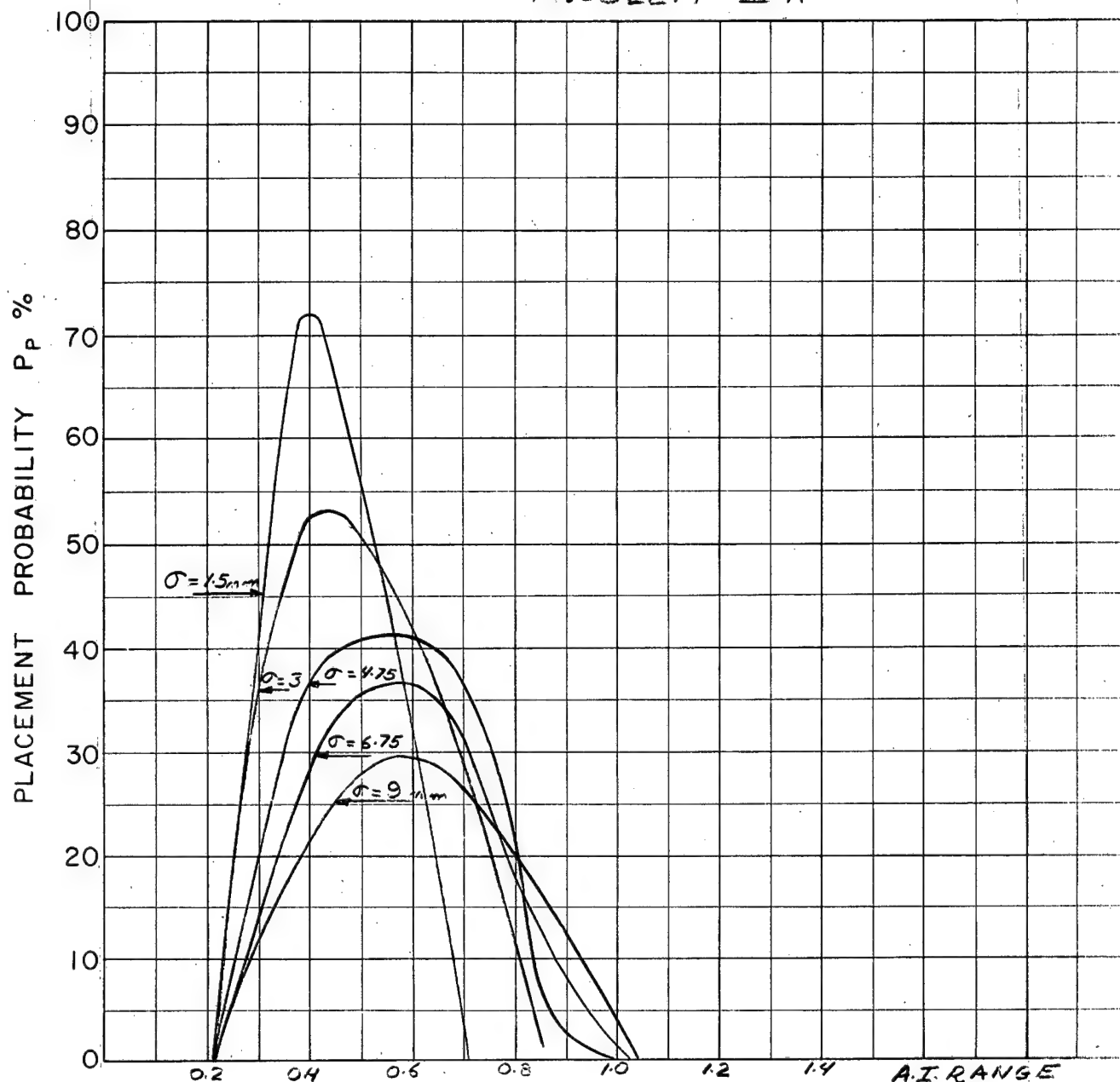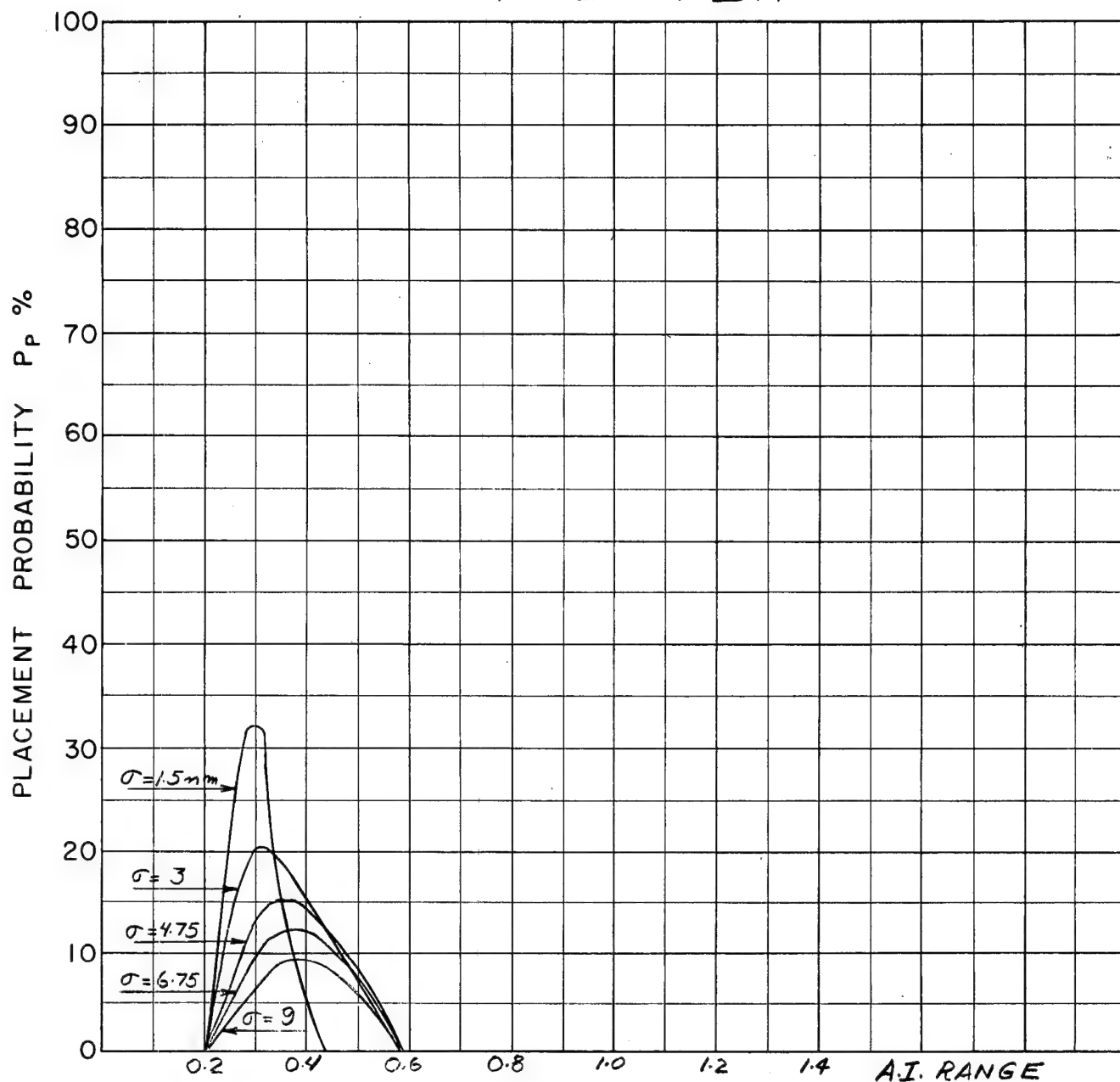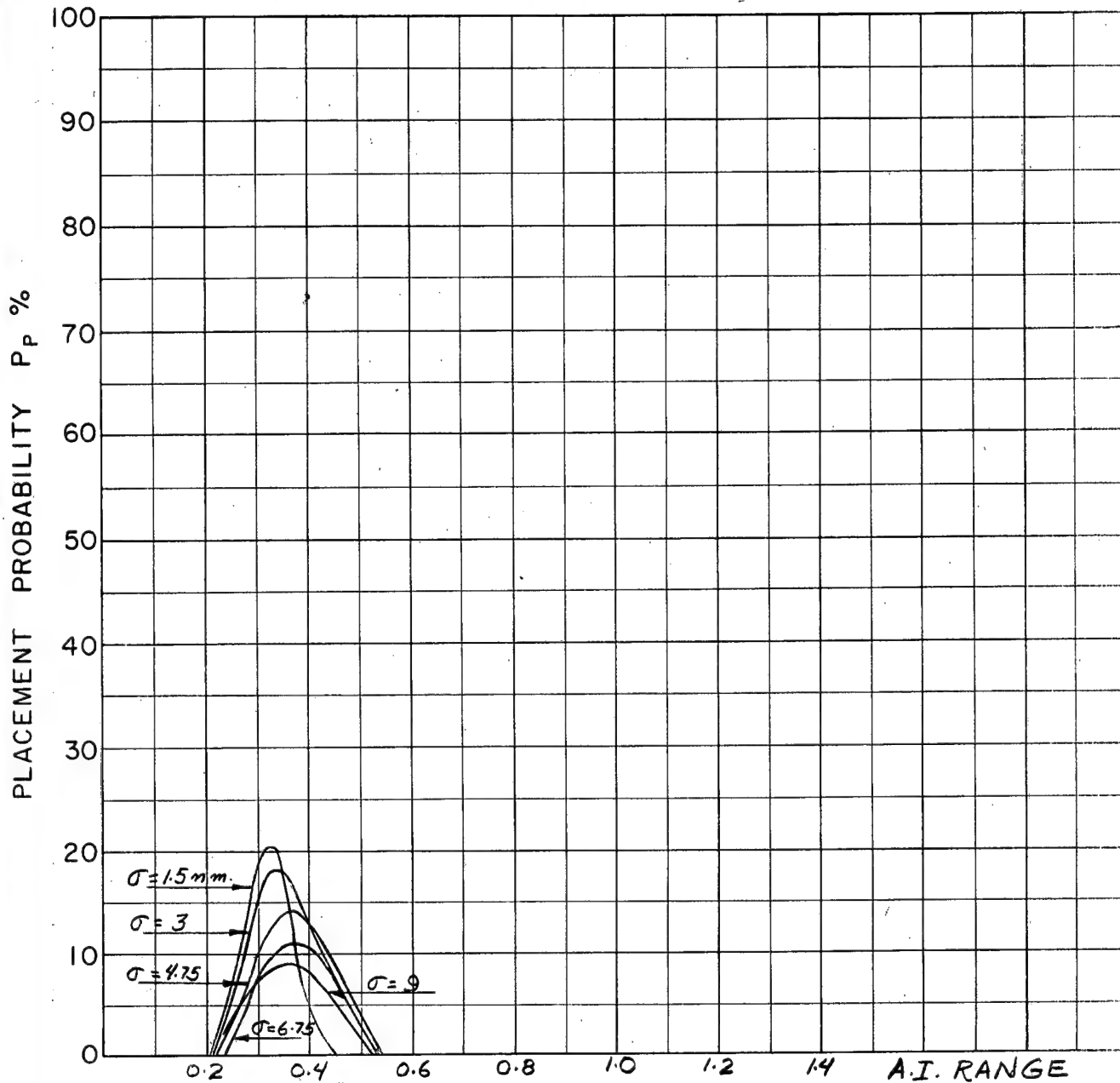σ OF G.C.I. ACCURACY: 5 Values
A.I. DETECTION RANGE AS FRACTION OF SPECIFICATION RANGE, S: Abscissa
A.I. DETECTION RANGE CONTOUR: Delta
ALTITUDE: $H_t = 60$ K
$H_{f_o} = 40$ K

APPENDIX 'B'

FIRE CONTROL STUDIES

by

J.A. Ockenden

1.0 INTRODUCTION

During Phase I of the CARDE Assessment Study of the CF105 Weapon System, ending in April 1957, a small group was formed for the study of Fire Control problems. Effort was concentrated on criticism of the RCAF Specification Air 7-6 and the RCA proposal DSG-105-1-5405 on the Electronic System, reported in CARDE Tech. Letter N-47-18. In addition, time was spent in monitoring and taking part in the 3-D and 2-D placement studies.

At the beginning of Phase II of the Assessment Study, the Fire Control Group was reduced in strength from two to one. Two new arrivals are expected in August, while the writer will be leaving at the end of September, so that the strength will be two again for the latter part of Phase II.

During the period under review, study of several problems involving the theory of fire control has been initiated. Two visits of two weeks each have been made by Mr. A.C. Stonell of the de Havilland Aircraft of Canada, Ltd., and with his assistance a program of fire control studies has been set up. It is hoped that six of the nine problems listed will be completed by the end of September, leaving three which involve analysis of results of the present REAC investigations. In addition, any further problems which arise as the work progresses will be studied in the latter part of Phase II.

The first three Quarterly Progress Reports on ASTRA I have been received from R.C.A. Several fire control studies are described, but in some cases the significance and validity of the conclusions are difficult to determine, because of lack of detail in the description of the study. Presumably the details of the various studies are included in separate reports, and some of these would be useful to CARDE.

2.0 FIRE CONTROL PROBLEMS

A list of nine topics has been drawn up, and these are enumerated, with brief notes concerning the present status and the form the studies are expected to take. The topics are:

    (a) A.I. Phase Navigation
    (b) Range of Instrumentation of Fire Control Computer parameters
    (c) Launch Zone Instrumentation
    (d) Minimum Information Study

(e) Snap-up Instrumentation
(f) The Effect of V_F changes in Horizontal Turns
(g) Choice of Attack Mode
(h) Navigation for Favoured-Aspect Firing
(i) Diving or Diving Turn Attacks.

2.1 <u>A.I. Phase Navigation</u> - A requirement of the RCAF Specification, Air 7-6, was three attack modes, namely lead collision, lead pursuit and optical. In their third quarterly, R.C.A. proposed that lead pursuit, required for missiles only, was in general inferior to lead collision, and hence would be deleted. It is felt at CARDE that whilst, in most cases, lead collision is adequate, there may be a significant number of attack situations in which lead pursuit would succeed where lead collision fails.

R.C.A. studies have shown that lead collision navigation is superior in requiring less time from lock-on to interception, and in demanding less turning at the end of the attack. However, there are some situations in which the lead collision equations become unstable. Theoretical work at CARDE has shown that in all cases considered to date, the equations as mechanized are satisfactory unless the range is opening. A positive range rate causes computed time to go to become negative, and the computer is not capable of handling this situation.

It remains, then, to determine the significance of attacks in which an opening range arises. The difficulty could be overcome by switching to lead pursuit navigation at about the maximum firing range, and there are other forms of navigation which might be used. Switching to lead pursuit is easily mechanized, but definite proof of the necessity is required before the case can be presented to R.C.A. The study will progress with the aim of determining this necessity.

An alternate mode of A.I. phase navigation is modified lead pursuit, i.e. lead pursuit with an artificially small value of missile velocity used to compute the early part of the attack course. It is not considered likely that this form of navigation would be better than lead collision, or lead collision plus lead pursuit, but a brief investigation would be in order if time permits.

A modification of lead collision was studied in a previous 2-D REAC investigation. Steering error polarity, in beam attacks, was used to determine the fighter rate of turn in correcting a vectoring error. It is felt that the results of this study should be re-examined, and the work extended if necessary. Also the way in which the computer could be modified to incorporate this mode of attack must be examined, if improvements in placement probability are significant.

2.2 <u>Range of Instrumentation of Fire Control Computer Parameters</u>- The two most critical parameters are range rate and time to go, so that this study is very closely allied to the previous one. At present there is no reason to suspect that the other parameters are not adequately instrumented, but a quick examination would be advisable. Problems 2.1 and 2.2 will be worked in conjunction.

2.3 <u>Launch Zone Instrumentation</u> = The launch zone mechanization
proposed by R.C.A. is very similar to that proposed for Sparrow II
by Douglas in Douglas-Bendix Sparrow II Quarterly Progress Report
SM-19564. A study has been initiated to check this instrumentation
from the point of view of the ASTRA System. In particular, the
use of lead collision steering to a launch zone of finite width
($R_{max}-R_{min}$) should be studied. A constant t_f value of 7 seconds
is to be used, with missile incremental velocity, and hence F pole,
a function of altitude, according to the empirical equation:

$$V_M = 615 + 82 \frac{p_{sl}}{p_s} \quad \text{(feet/second)}$$

where V_M is the average missile incremental velocity

and $\frac{p_s}{p_{sl}}$ is the pressure ratio.

The resulting firing ranges must be checked at all altitudes,
aspects and speed combinations for compatibility with the
mechanized R_{max} and R_{min} values. In addition, since firing is
permissible at times other than t_f (the ideal firing time),
aiming errors must be examined in detail.

The planning of this study is well advanced.

2.4 <u>Minimum Information Study</u> = A study is proposed on how best
use can be made of what information is available under E.C.M.
conditions. A considerable amount of work has already been
undertaken by the CARDE Systems Group along these lines, and the
planning and supervision of the continuation of this study will
remain within the Systems Group.

One suggested topic for study is the use of artificial values
of range and range rate to enable navigation to the expected
cross-over range. The fixed range value, equal to the expected
cross-over range would be used, and an estimated range rate value,
together with angle information from the jammer homer or IR sub-
system. The courses resulting from this method of approximate
navigation will be studied. Another possibility is the use of
the dead reckoning computer to assist in navigating to the
expected cross-over point. In addition, the way in which the I.R.
sub-system can be put to best use should be examined as a separate
topic.

2.5 Snap-up Instrumentation.- In the first R.C.A. Quarterly,
equations for snap-up instrumentation were presented . More
recently, it has been decided by RCA to delete the angle to
turn through and time to snap up computations. They will be
replaced by a nominal value of time to go at which snap-up
will be initiated. A value of 30 seconds to go is suggested,
but it is not clear what target and interceptor altitudes this
value refers to, or whether the time should be a function of the
altitudes.

.Preliminary investigations at CARDE, on paper and on the
REAC, have shown that the latest time at which snap-up can be
successfully initiated is an active function of altitude
difference, and also depends on target altitude. However, it
may prove satisfactory to use one large time to go value at
all altitudes and altitude differences.

The third R.C.A. quarterly gives equations for an artificial
F pole computation:

$$F^1 = F - \frac{g}{2} (T^2 - t_f^2) \ E_{ave}$$

where E_{ave} is a nominal value representing the average aircraft
pitch angle during snap-up. The use of F^1 in the computer, instead
of F, compensates for velocity slowdown, so that bank angle at
launch, and hence miss-distance, are appreciably reduced. No
value of E_{ave} is given, but it would appear to be in the vicinity
of 25°. The theory of this compensation will be examined further
at CARDE in the next quarter.

2.6 The Effect of V_F Changes in Horizontal Turns - Compensation
for velocity slowdown in snap-up attacks will be studied under
the problem of section 2.5. Closely allied to this problem is
the effect of velocity slowdown in horizontal turns. It has been
suggested that some form of compensation in the steering equations
or parameter values, allowing for the expected slowdown, might
reduce the time taken to correct initial vectoring errors, and
hence increase the probability of error reduction before launch.
A brief mathematical study of this problem will be conducted
in conjunction with the snap-up instrumentation investigation.

2.7. Choice of Attack Mode - The three attack modes under
consideration are climbing attacks, snap-up and dives. The
study falls into two parts: first, the tactical situations
under which each mode is advantageous must be determined, and
second, a mean must be established of deciding, in a particular
situation, which mode is best.

The results of the present 3-D REAC placement study will provide the necessary information for determining the advantages and disadvantages of each mode in all the expected attack situations. Thus the first part of the problem consists of analysing REAC placement charts and probability curves as they become available. If regions exist where one mode is significantly superior to another rules must be formulated which specify the better mode.

The decision of which attack mode is to be used, where there is a choice, may be made by the pilot or navigator, or it may be mechanized into the electronic system. A human engineering study would be required to determine whether automation is necessary.

2.8 Navigation for Favoured-Aspect Firing - The lethality study at CARDE will investigate, among other things the relationship between aspect angle of weapon approach to target and kill probability. If there are firing aspects which are significantly favourable, and these coincide with aspects that are preferable from the point of view of fighter placement, a significant increase in kill probability would result from firing at the favoured aspect. The ground control may attempt to vector the fighter onto a course difference which tends to result in firing at a particular target aspect. However, if there are vectoring errors at fighter lock-on, the lead collision navigation system will cause the fighter to turn onto a different course difference, and hence to fire from a different aspect. It may be possible to modify the navigation laws so that the fighter tends to remain at the desired aspect, for example by programming speed changes.

The results of the missile study and the placement studies will be analysed to determine the effect on overall kill probability of aspect at launch. If there are peaks in the relationship, the navigation laws should be examined to see whether modifications along these lines are practicable.

2.9 Diving or Diving Turn Attacks - The effects of altitude and altitude difference are being studied in detail in the 3-D REAC study. If the results show that there are situations where a lower fighter altitude would improve placement probability, a dive might be advantageous. Following a detailed analysis of the results of the present study, it may be decided that further REAC runs are required, with a dive instrumented. Then if dives are considered to be a useful form of attack, some thought must be given to the method of determining in which situations, at what time in the attack, and the practicability of instrumentation of dives in the aircraft electronic system.

3.0 SCHEDULING OF WORK

It is hoped that problems 2.1 to 2.6 will be completed by the end of September. The problems are mainly theoretical, although further REAC work may be required in problem 2.1. Problems 2.7 to 2.9 involve analysis of REAC results, and hence these problems will be started later in the program. It has been suggested that some thought should be given to the theoretical side of problems 2.7 to 2.9 before the analysis work is started, in case time does not permit this later. New concepts of fire control may be involved, particularly in problem 2.8, and perhaps in 2.7 and 2.9 if the scope of the study is interpreted liberally.

APPENDIX 'C'

MISSILE MISS DISTANCE DISTRIBUTIONS

by

C.J. Wilson and M.A. Meldrum

1.0 INTRODUCTION

There appears to be little information about the kill probability, P_K, to be expected from Sparrow II missiles when fired against high altitude targets. P_K for a missile depends upon:

a) target vulnerability;

b) warhead size and blast geometry;

c) performance of the fuzing system;

d) the distribution of trajectories about the target during the end phase, which may itself depend upon launching errors,

with a certain interdependence between these factors. Analysis is being attempted to obtain a quantitative estimate of P_K. Factor (d), the distribution of trajectories, is considered in this appendix. The effects of noise on the system have to be taken with account to obtain a solution.

2.0 METHOD

2.1 Literature Survey - A survey has been made of available references on radar scintillation and noise; this is described in Appendix "E". Reports on the effects of noise on missile performance have also been considered. In particular, a report on three-dimensional simulation of Sparrow II (Ref. 1) contains radial miss histograms for the case of attacks at 30,000 ft. against 900 ft/sec. targets.

However, it was concluded that insufficient information was available in these reports and the Douglas Aircraft Company have been asked for more data from their simulation work on Sparrow II. At the same time the system is being simulated on the analogue computer at CARDE in an attempt to obtain independent solutions.

2.2 <u>Simulation of Sparrow II to Date</u> - A simplified two-dimensional simulation of the missile has been made, based on information about simulation of the 1242C missile from a Douglas report (Ref. 2). Representations of the angular scintillation of the radar return from the target and of the internal system noise were included in the simulation. The target was non-evading at Mach number 0.85 at 50,000 ft. altitude.

The end course only of the missile attack has been studied so far. For this purpose the missile velocity was assumed constant. (This corresponds to a study of missile flight after the boost phase, with the effects of drag neglected). About 100 runs from each set of initial conditions were made in order to obtain the miss distributions.

R.M.S. miss values from this simulator work have been unrealistically small, of the order of 10 feet. It has been established that it is not possible to simulate the end course only, neglecting the previous history of the flight, because the more serious dispersions of trajectories of a constant-bearing missile take place during and immediately after the boost phase. If the end course only is studied a distribution of initial conditions must be chosen which corresponds to the deviations at the end of boost.

2.3 <u>Future Simulation</u> - There are two possible approaches to future work. These are:

a) simulation of the complete missile flight;

b) separate simulation of the boost phase and the end course, the former to provide initial conditions for the latter.

The relative merits of these methods are being assessed and preparations are being made for continuation of the work on the computer.

3.0 REFERENCES

1. Douglas Aircraft Company Report No. SM-18520 "Sparrow II Simulation Problems 2, Three-Dimensional Guidance Study" (Confidential).

2. Douglas Aircraft Company Report No. SM-27304 "Simplified Analog Simulation of 1242 C Missile" (Confidential).

APPENDIX 'D'

A STUDY OF RANGE FINDING BY MANEUVER

by

D.P. Flemming

1.0 INTRODUCTION

A problem that has received some attention during the CF105 study
at CARDE, is that which arises when the fighter radar equipment is
jammed in such a way as to make the direct measurement of target range
impossible by the usual means, so that one is led to considering the
feasibility of finding range indirectly from angular information. This
appendix discusses the principles of a particular method (T.L. N-47-12)
of doing this, namely by maneuvering the fighter aircraft in a prescribed
way, and noting the variation in the angular velocity of the line of
sight, from which the unknown range can be computed.

It is assumed that the fighter is engaging a single target aircraft[*],
which is transmitting a jamming signal, and that the type of jamming
signal is such that the fighter equipment is able to track in angle.

If the fighter travels uniformly on a straight line, then range
can not be determined uniquely on the basis of angular information
alone. Thus the fighter must perform some sort of maneuver. The
type that will be discussed here, and which in many ways appears to
be the most feasible, is a sinusoidal oscillatory path about the
collision course. One advantage of this type of maneuver is that the
mean course does not have to be exactly the collision course, since
effectively it is changes in the line of sight's angular velocity
that are made use of, rather than the value itself. Also, as will be
seen, these changes follow an approximately sinusoidal pattern and can
be correlated with the input maneuver, in order to eliminate much of
the noise in the line of sight angular velocity signal. This is important,
since the smallness of the angular velocity being measured (probably
less than 1 milliradian per second) makes noise a formidable problem.

[*]   Multiple targets are discussed in Section 5.

2.0 DESCRIPTION OF METHOD
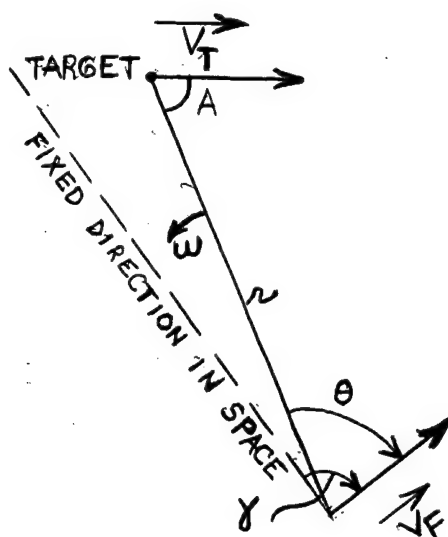
<u>Figure I</u>

Figure 1, above shows the primary kinematical variables used. These quantities are

$\overrightarrow{V_F}$ — fighter velocity vector

$\overrightarrow{V_T}$ — target velocity vector

γ — direction of $\overrightarrow{V_F}$, measured clockwise relative to space reference

θ — angle between $\overrightarrow{V_F}$ and line of sight

A — angle between $\overrightarrow{V_T}$ and line of sight

ω — anticlockwise angular velocity of line of sight in space

r — range, fighter to target.

The target is assumed to be travelling uniformly - target evasion will be discussed in Section 5.

At any time t,

$$r\omega = V_F \sin \theta - V_T \sin A, \tag{1}$$

$$\dot{r} = -\left[V_F \cos \theta + V_T \cos A\right] = -V_c . \tag{2}$$

The fighter is assumed to be close to the true collision course at all times. This assumption implies that ω is always small, that θ and A are slowly varying quantities, and that the closing speed V_c - defined in equation (2) - can be regarded as being constant.

From Figure 1:

$$\dot{\theta} = \dot{\gamma} + \omega \tag{3}$$

$$\dot{A} = -\omega. \tag{4}$$

If we differentiate equation (1), and substitute for $\dot{r}$, $\dot{\theta}$, and $\dot{A}$, using (2), (3), and (4), we obtain the equation

$$r\dot{\omega} - 2 V_c \omega = V_F \cos \theta . \dot{\gamma} \tag{5}$$

Suppose now that before an arbitrary time, say t = 0, the fighter's velocity direction γ is a constant, say γ_0, and that after t = 0,

$$\gamma = \gamma_0 + a \sin pt. \tag{6}$$

We could attempt to solve (5) exactly for ω under these conditions, but we assume that we are interested primarily in the solution during a short time interval after t = 0, lasting for one or two periods of equation (6), and that p is large enough so that r does not change much in this time. We thus find the approximate solution, assuming r to be constant - the other coefficients, - 2 V_c and $V_F \cos \theta$, can also be assumed constant because the fighter is approximately on a collision course.

We assume that just prior to t = 0 ω is equal to ω_0, which is small but not necessarily zero. It is easily demonstrated that ω does not undergo a discontinuity at t = 0, so we solve (5) with the initial condition $\omega = \omega_0$ at t = 0. The solution is

$$\omega = \left(\frac{a V_F . \cos \theta}{\sqrt{\left[1 + \frac{4 V_c^2}{r^2 p^2}\right]}} \right) \left[\sin pt - \frac{2 V_c}{r p} \cos pt \right]$$

$$+ \left[\omega_0 + \frac{2 a V_c V_F \cos \theta}{r^2 p \left[1 + \frac{4 V_c^2}{r^2 p^2}\right]} \right] e^{\frac{2 V_c t}{r}} \tag{7}$$

where "a" and p are defined by equation (6). 6517005-01

We have assumed that t is less than one or two periods of equation (6), and that r does not change much, relatively, in that time. This means that the quantities $\dfrac{V_c t}{r}$ and $\dfrac{V_c}{rp}$ remain small, so we can neglect squares and higher powers of these quantities. Thus equation (7) simplifies to

$$\omega = \frac{a\,V_F\cos\theta}{\mathcal{U}}\left[\sin pt - \frac{2V_c}{rp}\cos pt\right]$$
$$+ \left[\omega_0 + \frac{2\,a\,V_c\,V_F\cos\theta}{r^2\,p}\right]\left[1 + \frac{2\,V_c t}{r}\right] \qquad (8)$$

From equation (8) we see that ω consists of two parts, a sinusoidal part, (the first term), and a slowly varying part, - that is, slowly varying compared to the sinusoidal frequency. Furthermore the sinusoidal part has two components, an "in-phase" component, proportional to sin pt, which is in phase with the γ oscillation, as given by equation (6), and an "out-of-phase" component, proportional to cos pt.

Suppose that by some frequency and phase sensitive process, one can measure the amplitude of the in-phase sinusoidal part of the signal. Then the ratio of this amplitude to the amplitude "a" of the γ variation will be $\left[V_F\cos\theta\right]/r$. Thus, since V_F and θ can be measured directly, r can be computed. The problem of accuracy, however, might be critical. The fighter turning rate is limited, which sets a limit on the product ap. Also p has to be large enough so that r will not change too much during a cycle. Thus the amplitude "a" is limited. Under these conditions the ω signal is very small, so that one has to try to evaluate the accuracy of such a system in the presence of noise.

One method of extracting the in-phase signal is as follows. The sinusoidal oscillation of γ is performed from t = 0 to t = nT, where $T = \dfrac{2\pi}{p}$ and n is an integer, that is over an integral number of cycles.* The ω signal, during this interval, is multiplied by the function

$$S\ (t) = \text{sgn}\ (\sin pt) \qquad (9a)$$

* It will be shown in the following section that n = 1 is the best choice.

that is, the function which is + 1 when sin pt is positive, and - 1
when sin pt is negative - in other words a square wave of unit amplitude
in phase with sin pt. This product is then averaged over the interval
0-nT, that is one evaluates

$$M = \frac{1}{nT} \int_0^{nT} \omega(t) \, S(t) \, dt \tag{10}$$

If the amplitude of the in-phase component of ω is b say, this will
contribute $\frac{2b}{\pi}$ to the integral M. The out-of-phase component will
contribute zero to this integral, and the slowly varying part of the
signal will contribute approximately zero - a constant bias signal
would contribute exactly zero. This process will also have a filtering
effect on the noise in the ω signal. Thus one can say, approximately,

$$M = \frac{2b}{\pi} = \frac{2 \, a \, V_F \cos \theta}{\pi \, r} \tag{11a}$$

Hence r can be computed. The value of r thus computed would correspond,
approximately, to the time $t = \frac{nT}{2}$, that is midway in the interval

$0 - nT$.

A second method, which is a variation of the above, is that of
using the sine function itself for S(t), so that (9a) is replaced by

$$S(t) = \sin pt. \tag{9b}$$

In this case an amplitude b of the in-phase component of ω will
contribute b/2 to the integral M, so that (11a) is replaced by

$$M = \frac{b}{2} = \frac{a \, V_F \cos \theta}{2 \, r} \tag{11b}$$

The effect of noise in the ω signal will be to introduce a
statistical error into equation (11a), or (11b), and hence in the
computation of r. In the following two sections we will show this
error can be calculated, if the ω noise is assumed to be stationary
with a given spectral density.

3.0 EFFECT OF NOISE

We can assume that in the absence of noise the ω signal would be

$$\omega = \frac{a\, V_F \cos \theta}{r}\ \sin pt \tag{12}$$

since this is the only part of the ω signal which will contribute significantly to M. However when we also consider the noise, which may be of the order of magnitude of the signal itself or even larger, we must consider the resultant total signal, ω_1 say, as being

$$\omega_1 = \frac{a\, V_F \cos \theta}{r}\ \sin pt\ +\ h(t) \tag{13}$$

where $h(t)$ is the noise. Then the integral "M" as actually measured, which we shall call M_1, will be

$$M_1 = \frac{1}{nT}\int_0^{nT} \omega_1(t)\ S(t)\ dt = M + \frac{1}{nT}\int_0^{nT} h(t)\ S(t)\ dt \tag{14}$$

The quantity M above is the "no-noise" value of the integral, which is given by equation (11a) when the multiplying function $S(t)$ is the square wave function, and by (11b) when $S(t)$ is the sine function.

We wish to discuss the second term of (14)

$$E = \frac{1}{nT}\int_0^{nT} h(t)\ S(t)\ dt \tag{15}$$

E is the error in M due to noise and is a statistical quantity which has mean zero. We wish to know σ_E, the standard deviation of E.

We discuss the "square wave" case first. The "sine" case will be very similar, with only slight modifications.

The noise h(t) is assumed stationary with spectral density $N(f)$.

Let $S_n(t)$ be equal to $S(t)$ when t is between 0 and nT, and equal to zero for all other values of t. Then

$$n_{TE} = {}''Y{}'' = \int_{-\infty}^{\infty} h(t)\, S_n(t)\, dt \tag{16}$$

Since the noise h(t) is stationary, the mean square statistical average of Y, σ_Y^2, is equal σ_y^2, the mean square time average of the function y(t), where*

$$y(t) = \int_{-\infty}^{\infty} S_n\left[u - (t-nT)\right] h(u)\, du \tag{17}$$

Equation (17) can also be written as

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\, w(\tau)\, d\tau \tag{18}$$

where

$$w(\tau) = S_n(nT-\tau) = -S_n(\tau) \tag{19}$$

Thus y(t) represents what would be observed at the output of a filter, if the input were h(t) and the filter weighting function[2] were $w(\tau)$.

---

* The method employed here is a slight generalization of that used by Bennett and Fulton[1].

If the Laplace transform of $\omega(\tau)$ is $W(S)$, then the spectral density of $y(t)$, $P(f)$, is given by the equation

$$P(f) = N(f) \cdot X(f) \tag{20}$$

where

$$X(f) = \left| W(2\pi \, jf) \right|^2 \quad (j = \sqrt{-1}) \tag{21}$$

Then

$$\sigma_y^2 = \int_0^\infty N(f) \, X(f) \, df . \tag{22}$$

so that

$$\sigma_E^2 = \frac{1}{(nT)^2} \int_0^\infty N(f) \, X(f) \, df . \tag{23}$$

The function $X(f)$, when evaluated, is given by

$$X(f) = \frac{\sin^2(n\pi Tf) \tan^2 \frac{\pi T f}{2}}{\pi^2 f^2} \tag{24a}$$

For convenience in plotting $X(f)$, let

$$f_o = \frac{1}{T} = \frac{p}{2\pi} \tag{25}$$

which is the frequency of the input γ oscillation. Then (24a) may be written

$$f_o^2 X(f) = \frac{\sin^2\left(n\pi \frac{f}{f_o}\right) \tan^2 \left(\frac{\pi}{2} \cdot \frac{f}{f_o}\right)}{\left(\pi f/f_o\right)^2} \tag{26a}$$

For $n = 1$, (26a) reduces to

$$f_o^2 X(f) = \frac{4 \sin^4 \left(\frac{\pi f}{2 f_o}\right)}{\left(\pi f/f_o\right)^2} \tag{27a}$$

which is plotted as a function of f/f_o (the solid curve in Figure 2).

For the general case i.e. n a positive integer, if we assume that the noise h(t) has a uniform spectral density, N_0, then from (23) and (24a),

$$\sigma_E^2 = \frac{N_0}{(nT)^2} \int_0^\infty \frac{\sin^2 (n\pi T f) \tan^2 \frac{\pi T f}{2}}{\pi^2 f^2} \, df \tag{28a}$$

which, when integrated, works out to be

$$\sigma_E^2 = \frac{N_0 (2n - 1)}{2nT} \tag{29a}$$

For the case in which $S(t)$ is the sine function instead of the squared-wave function, the analysis is the same, except that equations (24a), (26a), (27a) and (28a) now become

$$X(f) = \frac{4T^2 \sin^2 \pi nTf}{\pi^2 \left[1 - T^2 f^2 \right]^2} , \tag{24b}$$

$$f_0^2 X(f) = 4 \frac{\sin^2 (\pi n f / f_0)}{\pi^2 \left[1 - f^2 / f_0^2 \right]^2} , \tag{26b}$$

$$f_0^2 X(f) = 4 \frac{\sin^2 (\pi f / f_0)}{\pi^2 \left[1 - f^2 / f_0^2 \right]^2} , \qquad (n = 1) \tag{27b}$$

$$\sigma_E^2 = \frac{N_0}{(nT)^2} \int_0^\infty \frac{4T^2 \sin^2 n\pi T f}{\pi^2 \left[1 - T^2 f^2 \right]^2} . \tag{28b}$$

Upon integration, equation (28b) reduces to

$$\sigma_E^2 = \frac{N_0}{4nT} . \tag{29b}$$

The quantity $f_0^2 X(f)$ (n = 1) as given by equation (27b) is plotted as the dashed curve in Figure 2.

In the following section we will discuss how the results as expressed by equations (29a) and (29b) are related to the relative error in computing the range r. However we can establish one point immediately. Equation (29b) shows that if the sine function is used for S(t), the variance of E is proportional to $\frac{1}{n_T}$, which is the inverse of the total integrating time, while from (29a) we see that if the square-wave function is used the variance is proportional to $\frac{1}{n_T}$ multiplied by a factor which increases with n. Thus in both cases if the total integrating time is fixed, there is no decrease in the error due to noise if n is increased from 1 to a larger integer. On the other hand the signal, which is proportional to "a", is decreased, because we have seen that there is a limit on ap, so that increasing p decreases the allowable value of "a". Thus the relative error is increased by increasing n above unity. Thus n = 1 is the best choice, and n will be assumed to be equal to 1 hereafter.

4.0 RELATIVE ERROR IN ν

Equations (11a), (11b), (29a) and (29b), will enable us to compute the relative error in $\frac{1}{\nu}$. This can be roughly assumed to be equivalent to the same error in r if the error is not too large. For example a 1% error in $\frac{1}{\nu}$ would be considered equivalent to a 1% in r. However caution must be employed, since the errors in this study might be more of the order of 20% or 30% or more, and furthermore a Gaussian error in $\frac{1}{\nu}$ produces a badly behaved error curve in r itself, one which has both mean and mean square values equal to infinity. The best approach seems to be to compute the mean square error for $\frac{1}{\nu}$, and assume the error in $\frac{1}{\nu}$ is Gaussian. Then one can easily compute the probability of the measured value of $\frac{1}{\nu}$ being between α and β say, which, if α and β are positive, will be the probability of the corresponding r value computed from the experiment being between $\frac{1}{\beta}$ and $\frac{1}{\alpha}$. Thus we shall consider $\frac{1}{\nu}$ as the quantity being measured.

We assume stationary noise with constant spectral density N_0. The relative error in $\frac{1}{\nu}$ is the same as that in M, since we can assume the errors in measuring V_F and cos θ are small. We obtain, from (29a) - with n = 1 - and (11a),

$$\frac{\delta M}{M} = \frac{1}{2} \sqrt{\frac{N_0}{2T}} \frac{\pi \nu}{aV_F \cos \theta} \qquad (30a)$$

in the square wave case.[*] Correspondingly in the sine function case we obtain

$$\frac{\delta M}{M} = \sqrt{\frac{N_0}{T}} \quad \frac{\nu}{a V_F \cos \theta} \tag{30b}$$

It is clear from (30a) and (30b) that the sine function case is preferable by a factor $\frac{\pi}{2\sqrt{2}}$, which is approximately 1.11.
We shall restrict ourselves to the sine function case hereafter, since the two differ only in this one factor, as far as our results are concerned.

If we assume "a" is chosen to give the maximum possible peak turning rate, $\dot{\gamma}_{max,}$ then

$$ap = \frac{2 \pi a}{T} = \dot{\gamma}_{max.} \tag{31}$$

so that

$$a = \frac{\dot{\gamma}_{max} T}{2 \pi} \tag{32}$$

and (30b) becomes

$$\frac{\delta M}{M} = \frac{2 \pi \sqrt{N_0}}{T^{3/2}} \quad \frac{\nu}{(V_F \cos \theta) \dot{\gamma}_{max}} \tag{33}$$

In order to simplify the estimation of $\delta M/M$ we make two further assumptions. We first assume $V_F = V_T$ so that

$$V_F \cos \theta = \frac{V_c}{2} \tag{34}$$

The other assumption is that the relative decrease in r during the integrating time T is always equal to a given constant, "q", that is

$$\frac{V_c T}{r} \equiv q = \text{a constant} \tag{35}$$

[*] δM is the root mean square error in M, previously called σ_E .

This of course can only be true approximately, or in the statistical sense, since in the individual case the fighter pilot does not know r and so can not adjust T to satisfy (35).

If we eliminate T and ($V_F \cos \Theta$) from (33) - using (34) and (35) we obtain

$$\frac{\delta M}{M} = \frac{4 \pi}{\dot{\gamma}_{max}} \sqrt{\frac{N_0 V_c}{rq^3}} = \frac{\delta\left(\frac{1}{r}\right)}{\left(\frac{1}{r}\right)} \qquad (36)$$

Thus equation (36) can be used to estimate the relative error in $\frac{1}{r}$.

The following numerical example is chosen for example only, and may not be completely representative of the CF105 system and tactical situation. Suppose we take

$$\dot{\gamma}_{max} = 4^o/sec = .0698 \text{ radian/sec}$$

$$V_c = 0.4 \text{ nautical mile/sec}$$

$$r = 75 \text{ nautical miles}$$

$$q = 0.1$$

$$N_0 = 0.5 \times 10^{-6} \text{ (radians/sec)}^2 \text{ per (cycle per second)} .$$

Then substitution in (36) gives

$$\frac{\delta\left(\frac{1}{r}\right)}{\left(\frac{1}{r}\right)} = .3$$

corresponding to a 30% standard deviation in the measurement of $\frac{1}{r}$.

Since by equation (36) the relative error is proportional to $r^{-\frac{1}{2}}$, one can expect the method to be more effective at long ranges than at short ranges, from the point of view of relative accuracy.

If we say, roughly, that the relative error in $\frac{1}{r}$ is equal to the relative error in r, and then multiply this by r, we get an estimate of the absolute error, in nautical miles. This is proportional to $r^{+\frac{1}{2}}$, and therefore decreases as r decreases, but not linearly.

5.0 FURTHER COMMENTS

It is evident that many simplifying assumptions have been made. This is because the ω noise was felt to be the critical factor, so that lesser effects have been neglected.

One serious problem could arise due to multiple targets, all jamming, or jamming in turn. This would cause the fighter radar to drift from one target to another and increase the ω noise significantly.

If the target evades, the effect is to add an additional forcing function to equation (5). Suppose $\vec{V_T}$ remains constant in magnitude but its direction changes with (anticlockwise) angular velocity $\dot{\phi}$, ϕ being a function of time. Then equation (5) becomes

$$r\dot{\omega} - 2V_c\,\omega = V_F \cos \theta\,\dot{\gamma} - V_T \cos A.\,\dot{\phi} \ .$$

Thus if the enemy had complete intelligence he could maneuver to cancel out the effect of the γ oscillation. This of course is unlikely. A steady target turn would probably appear as a slowly varying term in ω and be filtered out, at least partially. Further study should be done on this subject.

Perhaps the biggest unknown factor is the spectral density, of the ω noise. In view of this some experimental work is being considered, with the cooperation of the Missile Electronics Section of "G" Wing. The nature of the experiments would be to train a ground based MG-2 on a slowly oscillating target, causing a sinusoidal ω of maximum value around 1 milliradian per second or less, which conforms roughly with the amplitude expected in practice. The "square-wave" method, which is the easier one to instrument, would be used to attempt to detect the signal and measure its amplitude.

REFERENCES

1. R.R. Bennett and A.S. Fulton, "The Generation and Measurement of Low Frequency Random Noise", Journal of Applied Physics, Vol 22, No. 9, p 1187 (Sept 1951), (see Appendix on p 1190).

2. "Theory of Servomechanisms", M.I.T. Radiation Laboratory Series, Vol. 25.

FIGURE 2

SPECTRAL DENSITY MODIFYING FACTOR $X(f)$

$S(t)$ = Square Wave Function $f_0^2 X(f) = \dfrac{4 \sin^4\left(\frac{\pi f}{2 f_0}\right)}{\pi^2 \left(f/f_0\right)^2}$

$S(t)$ = Sine Function $f_0^2 X(f) = \dfrac{4 \sin^2\left(\frac{\pi f}{f_0}\right)}{\pi^2 \left[1 - f^2/f_0^2\right]^2}$
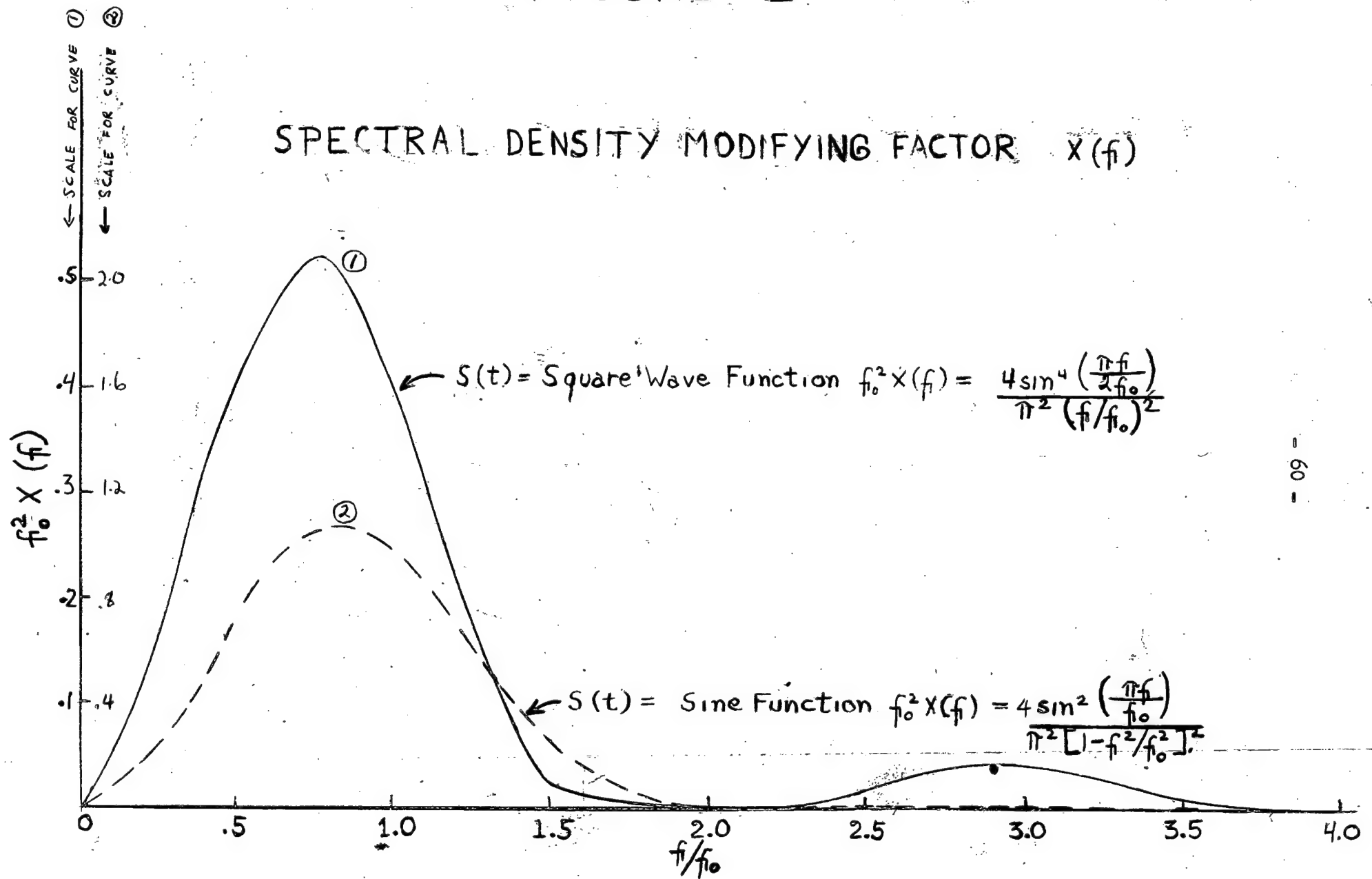
APPENDIX 'E'

The Effectiveness of a Constant-g Evading Turn

Executed by a Target Attacked by a Long-Range Rocket

by J. Cummins

The purpose of this appendix is to present the results of a short study on the effectiveness of a constant-g evading turn in bringing the target away from an explosion center.

Targets of Mach No: 0.92, 1, 1.5, 1.8 and 2 were considered with lateral accelerations ranging from 0.5g to 2g in 0.25g steps.

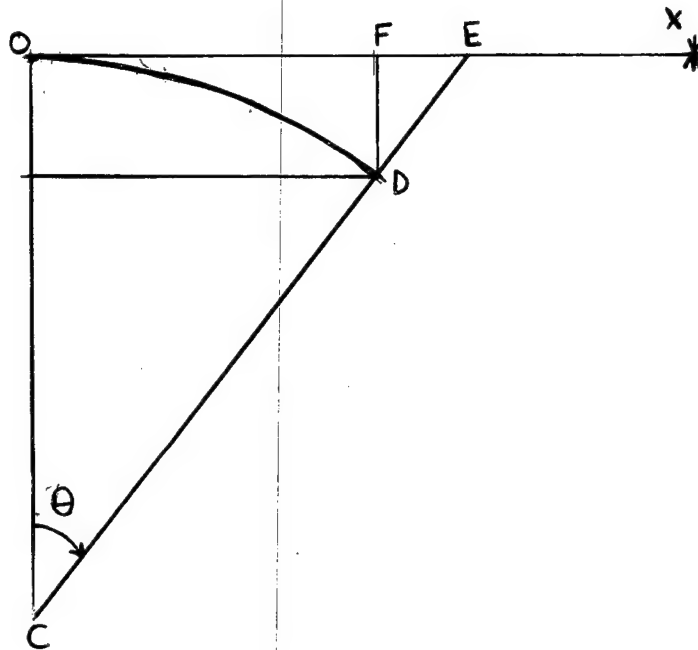Fig. E - 1 illustrates the geometry of the evading turn.

Figure E - 1

OE: represents the distance travelled by a non-evading target at a given Mach No.

OD: the trajectory of the evading target

X : the explosion center

It is required to find the distance XD.

$$= \frac{(971)^2}{32.2}\left[\left\{\frac{1}{2!} \cdot \left(\frac{32.2}{971}\right)^2 \cdot g\ t^2\right\} - \left\{\frac{1}{4!} \cdot \left(\frac{32.2}{971}\right)^4 \cdot \frac{g^3\ t^4}{M^2}\right\} \ldots\ldots\right]$$

From this expansion, it appears that the lateral acceleration is the most important factor in evaluating the effectiveness of target evasion and that target Mach number is of secondary importance.

The results of the study are shown in graphical form in Fig. E-2.

In table I are listed the average distances of the target from the explosion center for different values of lateral acceleration when the Mach No. is varied from 0.92 to 2.

The largest error made by using the quoted figures is also indicated.

The error should be added to the average value at interceptor Mach No. 0.92 and should be subtracted at Mach No. 2.

TABLE I

Lateral Acceleration in 'g's'	Distance from Explosion Center	Largest Error Made
.5	1100 feet	10 feet
.75	1192 "	16 "
1	1322 "	14 "
1.25	1540 "	18 "
1.5	1638 "	22 "
1.75	1816 "	28 "
2	1996 "	28 "

Let M be the target Mach No.

g, the lateral acceleration of the target in g's

Then $R = \dfrac{M^2 (971)^2}{32.2g}$ (feet)

$\theta = \dfrac{32.2g\ t}{971\ M}$ (radians) $F E = 971\ M_t t - R \sin \theta$ (1)

$FD = R (1 - \cos \theta)$ (2)

$XD = \sqrt{(EX + FE)^2 + (FD)^2}$

The distance FE is the difference of two large quantities. Greater accuracy will be obtained with less effort if the expression for FE is modified as follows:

$FE = .971\ M_t t - \dfrac{(971\ M)^2}{32.2\ g} \sin \dfrac{32.2\ g\ t}{971\ M}$

$= 971\ M_t t \left[1 - \dfrac{\sin \theta}{\theta} \right]$ where $\theta = \dfrac{32.2\ g\ t}{971\ M}$

$= 971\ M_t t \left[\dfrac{\theta^2}{3!} - \dfrac{\theta^4}{5!} + \dfrac{\theta^6}{7!} - - \right]$

By neglecting the term $971\ M_t\ \dfrac{\theta^4}{5!}$ an error is introduced of magnitude:

$\mathcal{E} = 0.1765\ \dfrac{g^4}{M^3}$

For the cases considered, the largest error will occur with $g = 2$ and $M = 0.92$. The error is then $\mathcal{E} = 0.1765 \times \left(\dfrac{16}{.78} \right) = 3.6$ feet, which can be neglected.

The expansion of $FD = R (1 - \cos \theta)$ in a series shows that the target Mach No; M, does not appear in the first order term.

$FD = \dfrac{(971\ M)^2}{32.2\ g} \left[1 - \cos \theta \right]$

$= \dfrac{(971)^2\ M^2}{32.2\ g} \left[\dfrac{\theta^2}{2!} - \dfrac{\theta^4}{4!} + \dfrac{\theta^6}{6!} - - \right]$

$= \dfrac{(971)^2\ M^2}{32.2\ g} \left[\left\{ \dfrac{1}{2!} \cdot \left(\dfrac{32.2}{971} \right)^2 \cdot \dfrac{g^2 t^2}{M^2} \right\} - \left\{ \dfrac{1}{4!} \cdot \left(\dfrac{32.2}{971} \right)^4 \cdot \dfrac{g^4 t^4}{M^4} \right\} \ldots \right]$

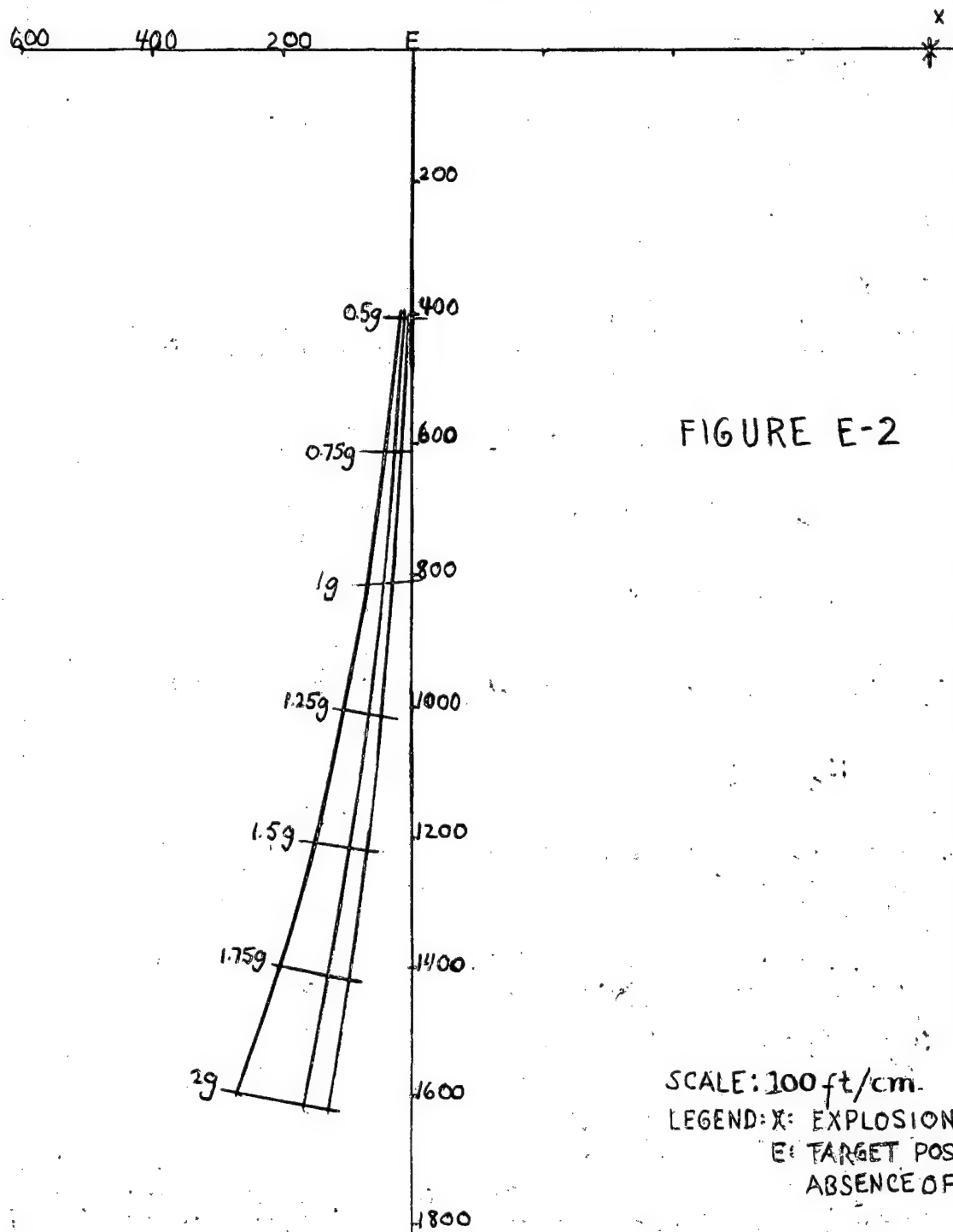# TARGET POSITION AT THE TIME
OF ROCKET EXPLOSION

FIGURE E-2

SCALE: 100 ft/cm.

LEGEND: X: EXPLOSION CENTRE
E: TARGET POSITION IN THE
ABSENCE OF EVASION

APPENDIX 'F'

LIAISON VISITS TO R.C.A.

Three liaison visits were made with R.C.A. Given below are outlines of the information discussed.

I

Visit to R.C.A.

1.0   A joint RCAF/CARDE party visited RCA establishments at Waltham and Camden on April 8, 9 and 10th, 1957. The object was to obtain information on the ASTRA I electronic system being developed for the CF105 aircraft.

The party was made up as follows:

D.Arm/E	–	S/L Young
D.S.E.	–	S/L Peek
	–	E. Leese
D.Phys.R.	–	W.W.H. Clark
D.R.T.E.	–	A. Walker
C.A.R.D.E.	–	R.S. Mitchell
		J.T. Macfarlane
		J.P. Regniere

Outlined below are the impressions of the C.A.R.D.E. party.

A. Visit to Waltham

2.0   The first two days, April 8 and 9, were spent at the Airborne Systems Laboratory of RCA, in Waltham, Mass. The agenda for the discussions is given below.

I. The role and organization of ASL in the Astra effort.

II. General review of past and future programs of ASL on the Astra Weapons system.

III. Canadian ground environment.

     A. Typical sectors. Mission profiles and the Marshalling point concept.
     B. Broadcast control and close control.

IV. Conversion barriers.

    A. As a function of interceptor altitude, target-interceptor altitude differential, interceptor and target velocities, off-course error.
    B. Analytic and simulator studies.
    C. Relationship to CARDE studies.

V. Interception tactics: tactical use of Astra I with Sparrow II armament.

    A. Lead-collision course vs. lead-pursuit.
    B. Blind region problems.
    C. Automatic and manual modes.
    D. Launcher extension.
    E. Ripple firing.

VI. Sparrow launch zones with Astra I.

    A. As a function of closing rate, launch velocity, altitude.
    B. Climbing, diving and snap-up attacks.
    C. Target maneuvers.
    D. Warhead and fuzing.
    E. Interceptor breakaway.

VII. Astra I system effectiveness in presence of enemy countermeasures.

    A. Electronic countermeasures.
    B. Target maneuvers.
    C. Mass raids.

3.0   The organization of RCA for the CF105 project was briefly outlined. A diagram illustrating the various responsibilities is shown in figure F-1.

4.0 Work to Date

    RCA work has concentrated on the worst case - that of higher altitude targets with MB-1 armament. Operational studies have been completed and navigational requirements determined for this case. Work is proceeding on the tactics and the equipment problems for Sparrow missiles. The interchange of detection range, ground environment accuracy and aircraft manoeuvrability has been emphasized. Dynamic studies of the computer/coupler/autopilot configuration have been carried out. The radome, look angle and blinding problems have been looked at briefly. Much of this work was carried out on the kinematic simulator.

5.0 Ground Environment Studies

(a) Mr. McCarthy of RCA introduced general studies which have been made without substantiated assumptions. A 300 n.m. square defence sector was postulated, with early warning some 800 n.m. distant. The sector was based on the Bomb Release Line.
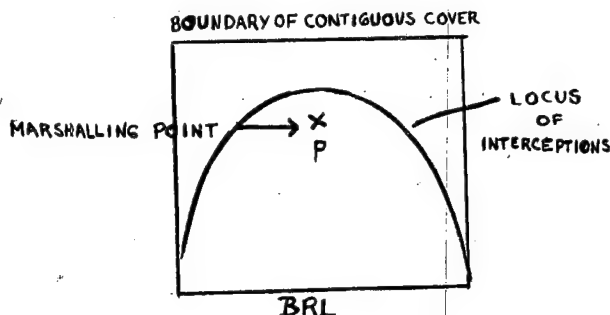
A Mach 2 target was assumed. It was shown that if the interceptor is at operational altitude at subsonic speed at a marshalling point (P in the figure) when the target enters the sector, the interceptions take place for all targets at least 217 nm before the BRL. The position of P marked on the sketch was optimized for the greatest average distance between the interception time and the BRL, this marshalling point is 214 miles from the BRL on the centre line of the sector.

The procedure is to scramble on E.W., proceed at M 1 to the marshalling point, and accelerate to M 2 when the target enters the sector. Assuming the worst case, with base at one corner of the sector, the interception mission time is 58 minutes at M 1 and 9 minutes at M 2.

(b) Mr. E. Blanchard of RCA described some models used in determination of requirements of navigation accuracy.

Ground environment accuracy is used as an input.

The Offset Point navigation technique is looked on with disfavour; some points which were brought up are

(i) longer course (penetration, fuel, reliability problems)

(ii) accuracy (intuitively the cumulative errors should be greater in this system. The generation of the O-P depends on target data which may not be accurately known; navigation errors add to this, as well as error in choosing the instant of initiation of turn).

(iii) ECM on data link (if data is lost the interceptor is on a wrong course for collision with the target).

(iv) Inefficient manoeuvres may be required (this broad generalization was not enlarged upon and only expresses an intuitive dislike for O-P).

(v) CARDE representatives pointed out that there may be a need to control aspect because of P_k considerations.

An interception model which the RCA group uses, drawn in impact point space, is as follows:

MISSILE FLIGHT

A/C CLIMB

DELAY FOR SNAP
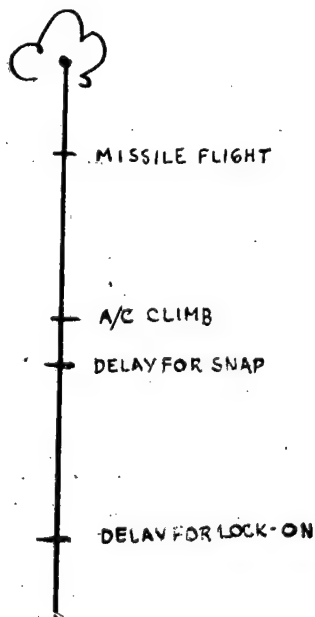
DELAY FOR LOCK-ON

working back along a trajectory from the impact point, times corresponding to missile flight, interceptor climb, time delay for snap, time delay for AI lock-on are marked; thus total time for interception is found. Distribution of starting points is found from combining probable ground control and navigation errors. If the initial path is incorrect, the minimum interception time is longer; a locus of possible initial points for a given impact point can be drawn.

To determine the probability of success, the required detection range is compared with the assumed detection probability of the AI radar. Curves of cumulated detection probability (P_D cum) are used. They were scaled from flight test values obtained by using an APQ 50 against a B-47. The values used for head-on probability, at 4000 ft/sec closing rate, are

50% @ 35 miles

80% @ 23 miles

90% @ 18 miles

for a 1 m^2 target. It was said that these figures were "very tentative".

In scaling of P_D the equation

$$P_D^* = 1 - (1 - P_D)^{nm^{\frac{1}{4}}}$$

is used where $n = \dfrac{\dot{R}^*}{\dot{R}}$ and $m = \dfrac{A_2}{A_1}$

$\dot{R}$ being relative closing rate for collision course and A the reflection area.

In estimating ground environment error effects, 2 mile position uncertainty, 15 second data rate, and 30 second (3-point) prediction were assumed. It was suggested that there were too many uncertainties in these assumptions for the results to be of any use.

This model is being used to compare direct attacks with offset-point attacks.

(c) A talk on data link was given by M.G. Slade of RCA.

For broadcast control the big problem was seen to be compatibility of the Canadian ρ/θ system with the US x/y coordinate system.

Close Control

The eight quantities required by the Sage system, and range of values, are

command	heading	-
command	altitude	0 - 12,800/128,000 ft
command	mach	(0 - 4)
attack	heading	-
attack	altitude	0 - 12,800/128,000
target	range	0 - 128 n.m.
target	bearing	-
time-to-go		0 - 128/4096 secs.

Wind is included in meteorological data, in the Sage system, and is not measured by the interceptor. If control is lost, it is proposed to follow last data given, and not to use the interceptor's computer and local data.

For return-to-base, the data given are return heading and altitude, range to base, and command Mach.

Data interval used is 30 seconds till 5 minutes before offset; then 15 secs to offset, with one data step being given after offset.

For <u>Broadcast Control</u> the data required and range of values needed are:

Target bearing	0 - 360
" range	0 - 512 n.m.
" heading	0 - 360
" speed	0 - 2560 kn.
" altitude	0 - 128 K ft.

Note that time division data link systems use as standard 8 bit quantity. 9 bits would be more useful but will probably not be used. In the b.c. mode, greater accuracy is expected if the wind data is inserted directly into the interceptor's computer.

6.0 Conversion Models

Although the AVRO aero data have been obtained, they have not yet been used in conversion studies, (what are called at CARDE placement probability studies). Fixed g constant speed turns in both horizontal and vertical turns for the cases treated have been

$$M_T = 2 \qquad M_I = 2$$

$$M_T = .9 \qquad M_I = 1.5$$

Brief look at faster targets

Barriers in target space for $30°$ angular placement error have been drawn for 2g and 4g interceptor turns. No target manoeuvre, no interceptor slow down, and no variation of missile heading error allowance with altitude, were used. Blanking of missile look by fuselage was not considered, nor does loss of AI look during corrective manoeuvre.

In determining probability of success overlays of P_D were used. Weighting of aspects was used as follows:

(0° head-on)	0° - 15°	20%
	15° - 30°	25%
	30° - 45°	20%
	45° - 60°	15%
	60° - 75°	10%
	75° - 90°	5%
	90° - 180°	5%

It was said that these were obtained by using the marshalling point concept. Probability of kill value for two missiles (provided by Douglas) was .75. Some results from this model are:

V_T	V_I	Vectoring Error 30°		More accurate vectoring
		N = 2g	N = 4g	10° 2g
f p s				
1000	2000	.72 coplanar	.74	.75
		.67 H=20K	.70	.69
2000	2000	.66	.68	.70
		.51	.59	.57

7.0 Displays

A brief summary of the moving airplane display was given by A.J. Weiss (Human Engineering). This has been described in the RCA proposal and progress reports. The "G" or preferred display gives an earth stabilized error dot and a flying airplane. The pilot flies the airplane onto the dot. The error signals in pitch and azimuth are the same rate signals which are fed to the autopilot. An auxiliary pitch angle scale is given. The "H" display uses pitch indication only instead of a pitch rate error signal.
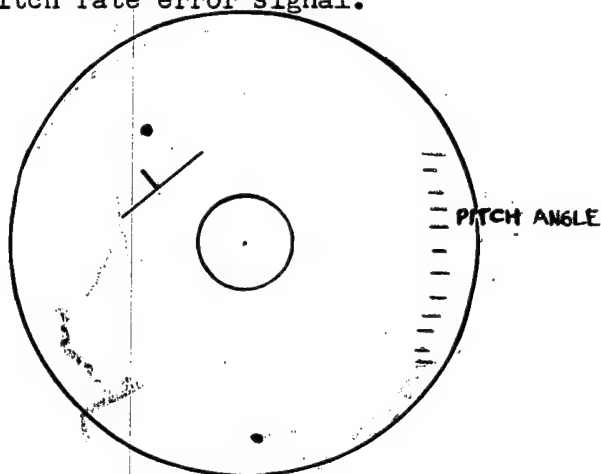
If the correction rates are correct the airplane stays on the dot.

A comparison of the E type display with these two new displays has been made using the simulator. Subjects used as pilot included "Duffers", engineers, and a group of RCAF pilots, some of whom were experienced in the use of the E display.

The case simulated was a beam attack on a M 1.5 bomber using M2 interceptor speed. Random bomber manoeuvre at three rates, $\frac{1}{2}$, 1, and $1\frac{1}{2}$ degrees/sec, was used. The "pilot" was required to track the target from Range = 100,000 ft. to Range = 10,000 ft. Each man made 120 runs; for groups of 30, using alternating display types. The "H" type was used only briefly since the "G" appeared invariably superior. Results are presented as graphs of % time the pilot is within 1° of the correct course.

Results show learning is slow on E display but fast on G. Tracking is never worse with the G display; with experienced E-pilot, it is not better however.

These results were quite impressively in favour of adopting the G display from a purely technical point of view. It was pointed out that change-over to a new system implies a large re-equipment and re-training problem, with, however, ultimate benefits in reduced training times.

8.0 Interception Tactics (D. Wellinger)

(a) RCA prefers a lead collision course. The following points were brought out.

(i) Where $M_T \geqslant M_I$ successful conversion is impossible with Lead Pursuit in some cases where Lead Collision succeeds (even for non-evading case).

(ii) In the manoeuvring target case tail-cone positions are more common, so that G-requirement is greater and speed may reduce. Lead pursuit course requires 2 g turn acceleration for a non-evading target and more for an evading target.

(iii) The missile heading error allowance in effect provides launch zone depth.

(b) F-Pole

Sparrow II missile allowable heading error is actually centred about Lead Pursuit position. (This statement is not exact). Lead angle requirement for the missile is less than that for a lead collision course at long range: as range reduces the lead angle enters the region of permissible values. This is best shown graphically:
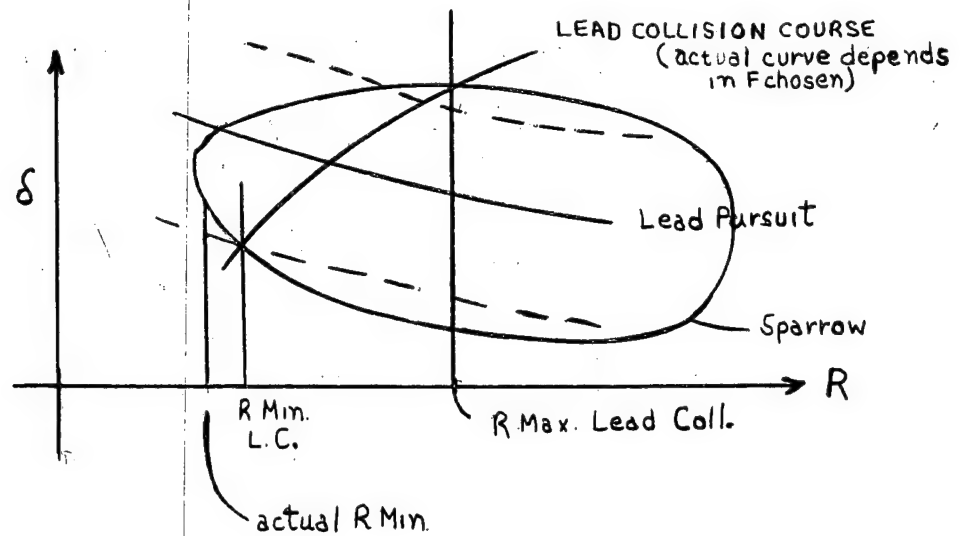
Launch conditions proposed are:

Range between limits R_{max}, R_{min}

$$\delta_{LP} \leq \delta \qquad \text{sparrow}$$

gimbal angle in elevation such that seeker must see target.

No mention was made of how Max and Min range were made to vary with aspect.

A 7 second missile flight time is used in the computations of lead angle. Both Lead Pursuit and Lead Collision error angles are computed.

$$\delta_{LC} = S_1 \left(R_{LS} - \frac{F}{T} \sin P_r \right)$$

$$\delta_{LP} = S_2 \left(R_{LS} - \frac{F}{T} \sin P_r \right)$$

$$\text{LP} \begin{cases} T = \text{Time to go} = \dfrac{R - F \cdot \cos P_r}{-\dot{R}} \\[2mm] F = \text{relative travel} \end{cases}$$

$$\text{LC} \begin{cases} \dfrac{F}{T} = V_M = \text{relative missile velocity} \end{cases}$$

For steering δLC is to be used. The firing indicator uses LP.

    (c) The proposed manual mode displays the two lead angles δLP and δLC.

        A circle of allowable Sparrow heading error around the δLP dot is given 5 seconds before T max.

9.0   On Tuesday morning, April 9, Mr. Walker of DRTE gave an exposition of his work on the ECM aspects of the ASTRA I system.

     The RCA work on range finding by manoeuvre has not yet been completed.

10.0   The launcher extension problem is considered to be serious by RCA because of the drag and consequent deceleration. The limiting factors on the time which the launchers must be extended are the firing delays which are $1\frac{1}{2}$ seconds before the first missile, $\frac{1}{2}$ second more for the second, (3rd and 4th may be at 2.5 and 3.6 seconds later, or .5 and 1 seconds later depending on the firing system adopted.

     It is reasonable to propose that extension take place at missile maximum range and not before. Lock-on of missile after launch would reduce the launching time required and ease the problem.

     The sudden drag increment causes a severe pitching moment. Although this may not be too serious with missiles, it may well render MB 1 attacks abortive.

11.0   RCA (McCarthy) queried

    (a) why the Take off fuel load stated in the RCAF Specification mission is less than 100% ?

    (b) why the M .2 placard speed ?

B. Visit to Camden

12.0   The agenda for Meeting at Camden, April 10th, was as follows:

9:30	IR Discussion	R.Merril, I Yurkevitch
		M. Petty
10:00	Tracking Loop Discussions	R. Aires

10:30   Antenna

       a) shape of antenna coverage R. Aires
       b) discussion of look angle S. Katten

11:00   Discussion of vertical & heading ref.   H. Halpern

11:30   Discussion of accuracy of velocity sensors H. Halpern

12:00   Lab tour.

13.0 IR Cells

Cell tests were Corona lab tests using NBS method.

(a) Spectral Response

(b) Time Constant (from frequency response assuming simple
RC equivalent)

(c) Responsivity Measurement $=\dfrac{\text{Voltage Output}}{\text{Power Input}}$

P_bS cells are measured at 20° and -78° C

P_bTe at -196° C in addition.

Several black body temperatures are used. Contours of sensitivity
are plotted in % of maximum by using a light spot.

14.0 IR Target Assumptions & Choice of Cell

Possible targets are:

(a) Gray body radiation (tail pipe)

(b) Plume radiation (exhaust gases)

(c) Aerodynamic heating.

(a) Black Body temperature of tail pipe for B-47 and B-52 is 500° -
650° C, corresponding to 4 or 5 microns.

(b) Plume radiation has peaks at 2.7 and 4.3 microns.
Plume radiation varies with altitude (of Kodak results on J-48
in flight showed that radiation at 40 K ft is only 12% of that
at sea level).

Range Contour is given in the following graph: (4 inch aperture,
500° C gases, 40 K ft altitude).

(c) Skin temperature peak is at 8 - 10 microns.

In forward hemisphere attacks (a) cannot be considered. The lead telluride cell is compared to the gold-doped germanium cell and these targets in the following graph:
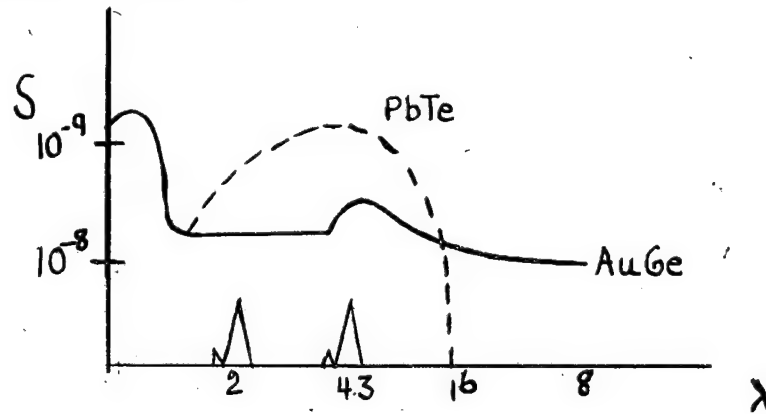
If plume radiation is to be the main target, the better cell is the P_bT_e cell which has an order higher sensitivity in the plume wavelength region.

Cooling and mounting provisions for the two cell types are the same so that change at later date is not prohibited.

15.0   IR   Installation

Present plans are for a small IR pod at the base of the radome, with look limits 20° up, 45° down, 70° in azimuth.

Problems being met in the design are those connected with slaving, and flexure.

(The original base of wind screen position provided too small a detector area).

16.0   Radar Antenna Look Angle Limits

The antenna design was discussed and drawings were examined. The design is well advanced, any request for a redesign would delay flight trials by several months.

The present tracking limits are:

72° up   45° down   ± 72° azimuth.

The az/el graph is a rectangle.

(These are practical limits, including effects of electrical and hydraulic limit stops).

The reasons for these limits and the overall problem of antenna design optimization were dealt with at length.

It was pointed out that the available 120° in elevation could be divided in any way between up and down.

II

Visit to C.A.R.D.E.

17.0   A representative of RCA visited CARDE on May 27 and 28. The main topic of conversation was the simulation studies being carried on at RCA and at CARDE with a view to establishing an understanding of the methods and assumptions employed at each establishment. General argument was reached between the two parties.

The following information was supplied by RCA on the equations being used for missile velocity, desired heading, and maximum and minimum launch ranges.

$$V_M = 615 + \frac{82}{P/PSL}$$

$$= \frac{2\,V_M}{V_I + V_M}\left[\frac{158\,(P/PSL) - 137\,(P/PSL)^2}{1 + 4.8\,(P/PSL) - 5(P/PSL)^2}\right]$$

$$K_1 = 11,500 - 27,400\ P/PSL \qquad\qquad P/PSL < .345$$

$$= 2,000 \qquad\qquad\qquad\qquad\qquad P/PSL > .345$$

$$K_2 = 18,700 - 5,800\ P/PSL$$

$$K_3 = 22,400 - 18,400\ P/PSL$$

$$R_{max} = \frac{-K_2 \dot{R}}{1000} + K_3$$

$$R_{min} = -9 \dot{R}_L + K_1 \qquad R_L = \dot{R}, \quad -\dot{R} < 1100$$

$$= -1100, \quad -\dot{R} > 1100$$

$$R_{max} < 36,000 \text{ ft.}$$

III

Visit to R.C.A.

18.0   A liaison visit was made to RCA Waltham on June 25, 1957, by R.S. Mitchell (CARDE) and S/L Peek (DSE). People seen at RCA were:

    D. Wellinger
    Earl Blanchard
    Warner Sievers.

19.0   Discussion centered around the work carried on at Waltham over the last quarter. This work centered around the following topics:

I Operational Analysis

    (a) Velocity Slow Down
    (b) Jump Factor
    (c) Radar Loop Angles
    (d) Target Maneuvers

II   Navigation

III Radar Detection

IV   Dynamic Analysis

V   E.C.M.

20.0 Operational Analysis

    Velocity slow down of the aircraft has been incorporated in the RCA simulation using approximations to the latest AVRO data. These approximations are most valid above 30,000 ft altitude and above Mach 1.3. Studies with this simulation to date have not turned up results which are markedly different from those at CARDE or previous conclusions reached at RCA.

20.1 Studies on the Sparrow missile indicated that the jump factor may be ignored if the launcher rails are inclined at 1^O to the aircraft axis. For safety reasons it is expected that the rail will be inclined at about 4 degrees. The jump factor may be allowed for either by feeding a compensation into the computer or by shrinking the error circle. Jump compensation is being made for the MB-1 Rocket.

It was mentioned that Sparrow III is considered as a third choice as a weapon and little work is being done on this missile in regard to the Arrow. However CW injection is still being provided.

20.2 Concerning the radar look angle limits further tactical studies have indicated similar results as previous in that it is prefereable to have more elevation angular look upward than downwards. Also Minneapolis-Honeywell have said that some instability may occur with large depression angles. In this regard reference was made to NACA report number RM A56K19.

20.3 Studies have been made on the effect of target maneuvers on time to go errors. Comments will be made on the subject in the next quarterly report.

21.0 Navigation

The general philosophy on initial phases of interception is that it is better to have the aircraft airborne and to altitude on GCI contact with the target; even if somewhat misplaced than to have the interceptor on the ground. Reference was made to a recent RCA report entitled:

"Interim Technical Report on Navigation Subsystem
for the ASTRA I Airborne Weapon System".

RCA again emphasized their preference for <u>not</u> having an offset point concept and gave the following reasons for this conclusion:

(a) The number of targets that can be intercepted with a high probability of successful conversion is greater for collision course tactics than for offset point tactics.

(b) There is increased error (perhaps instability) in going through on offset point. These errors are due both to ground environment accuracy and to errors in flying the aircraft.

(c) It is considered that there is little difference between P_{dc} for a specific aspect angle of approach and that resulting a collision course.

(d) Complexity and operational difficulties.

(e) If target knows that the interceptor is using an offset
point technique it can maneuve so that this procedure may
be very difficult or impossible.

(f) Penetration

(g) Fuel consumption

(h) If offset point concept is used to geographical coverage
is reduced.

22.0 Radar Considerations

Work has been done on considering time and frame size of the
search pattern. It would seem that a rectangle 40° x $13\frac{1}{2}^\circ$ is about
optimum size. Studies are in progress to determine how uniformly
this rectangle is pointed. Reference was made to a Hughes Aircraft
Report:

> "Prediction of Radar Detection Range from
> Flight Test Data" Tech. memo 379.

23.0 Dynamic Analysis

It was stated that aided tracking was found to reduce the error
by a factor of 5. It was recommended that this be used in addition
to error by pass.

An adjoint system analysis has been started to determine the
optimum smoothing for the fire control system when off course errors,
target maneuver and noise are considered. The noise was taken as
having an RMS of 30 feet at the target and an autocorrelation function
of:

$$\varphi_{ii}(\gamma) = 10^6 \left(\frac{\sigma}{R}\right)^2 e^{-\alpha(\gamma)} \qquad mils^2$$

where σ = RMS noise in $mils^2$

α = bandwidth

= 10 rads/sec.   (it was stated that perhaps this
should be 25 rather than 10).

The mean square of angular scintillation was found by evaluating the
autocorrelation function at γ = 0. For a range of 30,000 ft. this
value is one mil squared.

24.0 E.C.M.

It was agreed by RCA that in the multiple aircraft situation it was doubtful that the more sophisticated types of jammers could be used. However strong consideration should be given to these appliances because:

(a) they are relatively small (an X-band jammer has been built which weighs less than 100 lbs.);

(b) great effort and money is being expended in this direction in the U.S.

RCA considers that their system of AGC's is sufficiently versatile to prevent saturation of the amplifiers by noise jamming. It was not known at Waltham how much testing of breadboard equipment had been carried out at Camden.

It was stated that at present the AI radar does not yield sufficient information to permit angle track. However effort is being directed to accomplish this mode of attack.

In the semi-passive ranging mode it is assumed that there are delays in the repeater jammer's operation so that some range information may be obtained. The assumption is also made that lock on has been accomplished prior to jamming so that the range gate follows in an appropriate fashion during silent periods.

It was thought that random frequency amplitude modulation of the jammer should be within 5 to 10 cycles of the conical scan frequency of the AI to break angle track.

25.0 Fire Control Consideration

Discussion took place on the possibility of encountering negative time to go. It was noted that this situation did not normally arise due to the definition of T as time to impact. However if λ changes sign the computer fails. It was also noted that the system is designed for a high closing rate rather than a low.

In regard to compensation for velocity slow down it was not thought that coaltitude maneuvers produced sufficient slow down to necessitate compensation. It was RCA's philosophy to instrument for the "usual" cases rather than the unusual and the situations demanding high maneuvers were in the minority.

In the snap-up mode the compensation was made more from a desire to have the aircraft in level flight at firing rather than for corrections to time of flight calculations.

26.0 Future Work

In RCA's future simulation work emphasis will be on stability and error problems rather than analysis of tactical situations. It is hoped that the human engineering aspects of ECM can be studied on the simulator and possibly some attention given to mass raids.
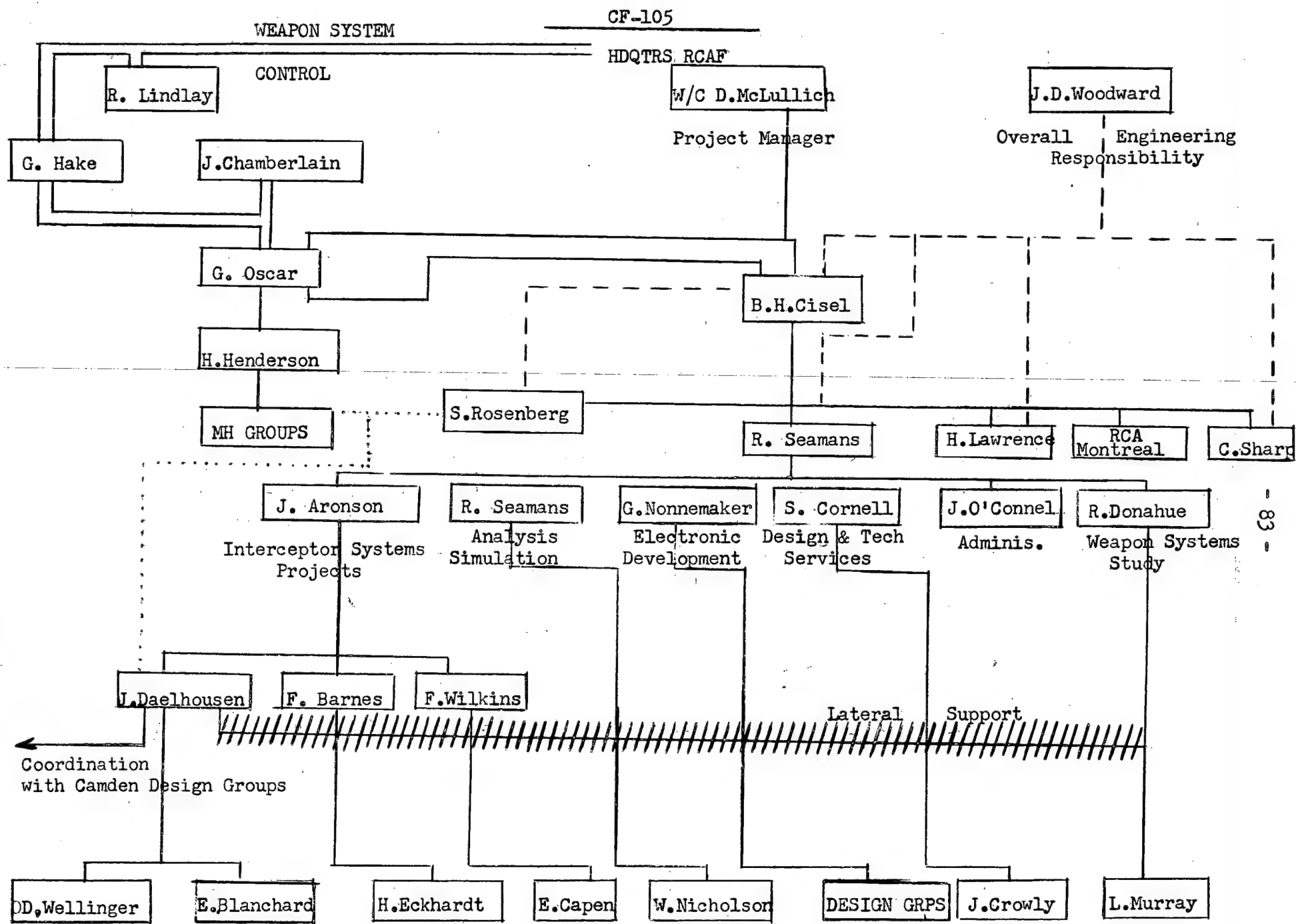
CF-105

WEAPON SYSTEM CONTROL / HDQTRS RCAF

FIGURE F-1

APPENDIX 'G'

OTHER RANGE-FINDING METHODS

by

F.W. Slingerland

Two other methods of passive range-finding in addition to that discussed in Appendix D have been evolved; both of which show considerable promise. Both give an expression for the time to go if on or near an aircraft collision course.

1.0 RATE MEASUREMENT

A fundamental property of combinations of two straight line courses is that:

$$R^2W = \text{Constant}$$

where R = range

W = angular rate of line of sight.

Differentiating and dividing by R gives

$$R\dot{W} + 2\dot{R}W = 0$$

Therefore $-\dfrac{2W}{\dot{W}} = \dfrac{R}{\dot{R}} = T$ the pseudo time to go.

The equation obviously has no solution when exactly on collision course, and practically speaking, no solution when very near a collision course due to the difficulty of measuring small w and ẇ . Hence T is only a pseudo time to go. However it appears possible to choose a value of this T at which successful launch can be achieved at nearly all aspects against a straight flying target. Future investigation will determine the shape of the T contours under various kinematic conditions (fighter attack mode and target evasion) and compare these with the missile launch contours.

2.0 POWER MEASUREMENT

For constant jammer power output the jamming power received by the fighter varies with range as follows:

$$P = \frac{\text{Const.}}{R^2}$$

taking logs and differentiating gives

$$\frac{\dot{P}}{P} = -2\frac{\dot{R}}{R}$$

Hence $-2\dfrac{P}{\dot{P}} = \dfrac{R}{\dot{R}} = T$ the pseudo time to go.

Thus by measuring received power and power rate an approximate launch time may be determined. This procedure is applicable to collision or non collision courses and appears to be useful even in the presence of fighter and target maneuvers. It is vulnerable to very slow power modulation by the jammer, but heavy smoothing can be used to remove higher frequency components since the true variation of P and $\dot{P}$ is very slow. Future work will include the construction of T contours under various conditions, and a study of the effect of multiple targets and power modulation.

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